A characterization of a class of 2-groups of order $2^{2(n+1)}$ by their defining relations

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For fixed $n \geq 3$ there are $9 \cdot 4^{n-1} + 32$ groups of order $2^{2n+1}$ which can be represented in the form $G = (C_{2^n} \times C_{2^n}) \rtimes C_2$ ([1]). Now we generalize this case to describe all groups of order $2^{2(n+1)}$ ($n \geq 3$) which can be represented in the form $G = (C_{2^n} \times C_{2^n}) \rtimes C_4$, i.e.,

$$G = \langle a, b, c \mid a^{2^n} = b^{2^n} = c^4 = 1, \ ab = ba, \ c^{-1}ac = a^pb^q, \ c^{-1}bc = a^rb^s \rangle,$$

where $p, q, r, s \in \mathbb{Z}_{2^n}$. We proved that there are

$$640 \ (\text{if } n = 3) \quad \text{and} \quad 12 \cdot 4^n + 512 \ (\text{if } n \geq 4)$$

such kind of groups and all these groups are described by their defining relations.

References