## ■ Ortogonaalteisendus, omaväärtused omavektorid

$$A = \begin{pmatrix} 0 & 1 & -3 & 0 \\ 1 & 0 & 0 & -3 \\ -3 & 0 & 0 & 1 \\ 0 & -3 & 1 & 0 \end{pmatrix}; \quad X = \{x1, x2, x3, x4\}; \quad Y = \{y1, y2, y3, y4\};$$

```
L = A - λ IdentityMatrix[4];
l = Solve[Det[L] == 0, λ]
Table[L /. l[[k]], {k, 1, 4}].X;
Table[%[[i, k]] == 0, {i, 1, 4}, {k, 1, 4}];
Table[Solve[%[[k]], X][[1]], {k, 1, 4}];
Om = (X /. %) /. x4 → u
% // MatrixForm
T = Simplify[Transpose[Table[1/Sqrt[Om[[k]].Om[[k]]] Om[[k]], {k, 1, 4}]], u > 0]
T.Y;
Table[X[[k]] → %[[k]], {k, 1, 4}]
Expand[Simplify[X.A.X]]
Simplify[% /. %%, u > 0]
Transpose[T].A.T // MatrixForm
```

$$\{\{\lambda \to -4\}, \{\lambda \to -2\}, \{\lambda \to 2\}, \{\lambda \to 4\}\}$$

$$\begin{pmatrix} -u & u & -u & u \\ u & u & u & u \\ u & -u & -u & u \\ -u & -u & u & u \end{pmatrix}$$

$$\left\{\left\{-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}, \left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}\right\}$$

$$\left\{x1 \to -\frac{y1}{2} + \frac{y2}{2} + \frac{y3}{2} - \frac{y4}{2}, \; x2 \to \frac{y1}{2} + \frac{y2}{2} - \frac{y3}{2} - \frac{y4}{2}, \right.$$
$$\left. x3 \to -\frac{y1}{2} + \frac{y2}{2} - \frac{y3}{2} + \frac{y4}{2}, \; x4 \to \frac{y1}{2} + \frac{y2}{2} + \frac{y3}{2} + \frac{y4}{2}\right\}$$

$$2\,x1\,x2 - 6\,x1\,x3 - 6\,x2\,x4 + 2\,x3\,x4$$

$$-4\,y1^2 - 2\,y2^2 + 2\,y3^2 + 4\,y4^2$$

$$\begin{pmatrix} -4 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$