1.7 Maatriksvõrrandi lahendamine

Ül 1.13. Lahendada maatriksvõrrand

$$\begin{pmatrix} -2 & 1 & 3 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} X = \begin{pmatrix} 23 & 25 & 27 \\ 2 & 4 & 6 \\ 9 & 9 & 9 \end{pmatrix}.$$

Ülesandes antud maatriksvõrrandiga $AX = B$ seotud maatriksid on

$$A = \begin{pmatrix} -2 & 1 & 3 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \quad \text{ja} \quad B = \begin{pmatrix} 23 & 25 & 27 \\ 2 & 4 & 6 \\ 9 & 9 & 9 \end{pmatrix}.$$

Leiame maatriksi A elementidele a_{ij} vastavad alamdeterminandid A_{ij}:

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}, \quad A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}, \quad A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ -1 & -1 \end{vmatrix},$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix}, \quad A_{22} = (-1)^{2+2} \begin{vmatrix} -2 & 3 \\ -1 & 2 \end{vmatrix}, \quad A_{23} = (-1)^{2+3} \begin{vmatrix} -2 & 1 \\ -1 & -1 \end{vmatrix},$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix}, \quad A_{32} = (-1)^{3+2} \begin{vmatrix} -2 & 2 \\ 1 & -1 \end{vmatrix}, \quad A_{33} = (-1)^{3+3} \begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix}.$$

Moodustame nendest elementidest maatriksi A adjungeeritud maatriksi

$$\tilde{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} = \begin{pmatrix} 3 & -5 & -7 \\ -1 & -1 & 1 \\ 1 & -3 & -5 \end{pmatrix},$$

ja leiame selle maatriksi determinandi

$$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = (-2) \cdot 3 + 1 \cdot (-1) + 3 \cdot 1 = -4.$$

Otsitav pöördmaatriks on

$$A^{-1} = \frac{1}{|A|}\tilde{A} = -\frac{1}{4} \begin{pmatrix} 3 & -5 & -7 \\ -1 & -1 & 1 \\ 1 & -3 & -5 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -3 & 5 & 7 \\ 1 & 1 & -1 \\ -1 & 3 & 5 \end{pmatrix}.$$

Maatriksvõrrandi $AX = B$ lahendiks on maatriks

$$X = A^{-1}B = \frac{1}{4} \begin{pmatrix} -3 & 5 & 7 \\ 1 & 1 & -1 \\ -1 & 3 & 5 \end{pmatrix} \cdot \begin{pmatrix} 23 & 25 & 27 \\ 2 & 4 & 6 \\ 9 & 9 & 9 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 4 & 8 & 12 \\ 16 & 20 & 24 \\ 28 & 32 & 36 \end{pmatrix}.$$

Otsitav maatriks on

$$X = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

Ül 1.14. Lahendada maatriksvõrrand

$$\begin{pmatrix} 1 & -4 \\ 1 & -3 \end{pmatrix} X \begin{pmatrix} 1 & 1 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} -3 & -8 \\ -2 & -2 \end{pmatrix}$$

Ülesandes on antud maatriksvõrrand $AXB = C$. Otsitava maatriksi X leidmiseks tuleb seda võrrandit korrutada vasakult maatriksiga A^{-1} ja paremalt B^{-1}, seega $X = A^{-1}CB^{-1}$.

Teist järku ruutmaatriksi A korral

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \Rightarrow \quad A^{-1} = \frac{1}{|A|} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}.$$

Järelikult

$$A = \begin{pmatrix} 1 & -4 \\ 1 & -3 \end{pmatrix} \quad \Rightarrow \quad A^{-1} = \frac{1}{-3+4} \begin{pmatrix} -3 & 4 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ -1 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & 1 \\ -1 & 4 \end{pmatrix} \quad \Rightarrow \quad B^{-1} = \frac{1}{4+1} \begin{pmatrix} 4 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 4 & -1 \\ 1 & 1 \end{pmatrix}.$$

Otsitava maatriksi X leidmiseks arvutame korrutise

$$X = \frac{1}{5} \begin{pmatrix} -3 & 4 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -3 & -8 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 1 & 16 \\ 1 & 6 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 20 & 15 \\ 10 & 5 \end{pmatrix}.$$

Järelikult maatriksvõrrandi lahend on

$$X = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}.$$