Exercises of Higher mathematics I

Homework 1

1. Find the domain of the function $y = \log(-x) + \dfrac{1}{x+5}$.
2. Find the domain of the function $y = \sqrt{3-x} + \arcsin \dfrac{3-2x}{5}$.
3. Find the domain of the function $y = \sqrt{\sin x} + \sqrt{16-x^2}$.
4. Find the domain of the function $y = \ln \dfrac{x-5}{x^2-10x+24} - \sqrt[3]{x+5}$.
5. Find the range of the function $y = 1 - 2\sin x$.
6. Find the domain of the function $y = \sqrt{3+2x-x^2}$.
7. Find the inverse function of the function $y = x^2 - 4x + 3$.
8. Find the inverse function of the function $y = \dfrac{2^x}{1+2^x}$.
9. Find the inverse function of the function $y = 1 - \log(3+e^x)$.
10. Find the inverse function of the function $y = 4\arcsin\sqrt{1-x^2}$.
11. Transform the function $\log_2 y - \log_2(x-1) = 3$ to explicit form.
12. Transform the function $(1+x)\cos y - x^2 = 0$ to explicit form.
13. Draw the graph of the function $y = |x| - x$.
14. Is the function $y = x - \dfrac{x^3}{6} + \dfrac{x^5}{120}$ even, odd or neither?
15. Is the function $y = x(5^x - 5^{-x})$ even, odd or neither?
16. Is the function $y = x^4 - 2x^3 + x$ even, odd or neither?
17. Is the function $y = x \cdot \ln \dfrac{1-x}{1+x}$ even, odd or neither?
18. Find $f\left(\dfrac{1+x}{1-x}\right)$, if $f(x) = \dfrac{1+x}{1-x}$.
19. Find $f\{f[f(1)]\}$, if $f(x) = x^2 - 1$.

Homework 2

In exercises 20. - 39. evaluate the limits.

20. $\lim\limits_{x\to 3} \dfrac{x^2-x-6}{x^2-9}$
21. $\lim\limits_{x\to \frac{1}{2}} \dfrac{8x^3-1}{6x^2-5x+1}$
22. $\lim\limits_{x\to 4} \dfrac{x^3-2x^2-8x}{x^2-x-12}$

23. $\lim_{x\to 2}\left[\frac{1}{x(x-2)^2}-\frac{1}{x^2-3x+2}\right]$

24. $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

25. $\lim_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49}$

26. $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

27. $\lim_{x\to 0}\frac{x^2}{1-\sqrt{1-x^2}}$

28. $\lim_{x\to\infty}\frac{(x+1)(x+2)}{2x^2}$

29. $\lim_{x\to\infty}\frac{10+x^5}{1-2x^5}$

30. $\lim_{x\to\infty}\frac{1-2x^2+3x^4}{1+2x^3}$

31. $\lim_{x\to\infty}\frac{x^{99}-1}{x^{100}+1}$

32. $\lim_{x\to 0}\frac{\tan 2x}{\sin 5x}$

33. $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin 2x}$

34. $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)$

35. $\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\tan x$

36. $\lim_{x\to\infty}\left(1+\frac{4}{x}\right)^{\frac{x}{2}}$

37. $\lim_{x\to\infty}\left(\frac{x^2+2}{x^2-1}\right)^{x^2}$

38. $\lim_{x\to\infty}\left(\frac{2x-3}{2x+1}\right)^{\frac{x-1}{2}}$

39. $\lim_{x\to 0}(1+x)^{\frac{2}{x}}$

40. Using the definition of the derivative, prove that $(\sqrt{x})'=\frac{1}{2\sqrt{x}}$

41. Using the definition of the derivative, prove that $\left(\frac{1}{x}\right)'=-\frac{1}{x^2}$

Homework 3

In exercises 42. - 50. find the derivative of the function and simplify, if possible.

42. $y = \dfrac{1 - x^2}{x^2 + x^3}$

43. $y = \log_3(x^2 + 2x + 4)$

44. $y = x \cdot 10^{\sqrt{x}}$

45. $y = \ln\left(x + \sqrt{1 + x^2}\right) - \sqrt{1 + x^2}$

46. $y = \sqrt[11]{9 + 6\sqrt[5]{x^9}}$

47. $y = \ln(e^x \cos x + e^{-x} \sin x)$

48. $y = \dfrac{1}{2}(3 - x)\sqrt{1 - 2x - x^2} + 2 \arcsin \dfrac{x + 1}{\sqrt{2}}$

49. $y = \dfrac{3x^2 - 1}{3x^3} + \ln\sqrt{1 + x^2} + \arctan x$

50. $y = \dfrac{\sin^2 x}{1 + \cot x} + \dfrac{\cos^2 x}{1 + \tan x}$

51. Evaluate $z'(0)$, if $z(t) = \left(\sqrt{t^3} + 1\right) t$.

52. The angle of rotation α of the belt drive depends on time as $\alpha = t^2 + 3t - 5$. Evaluate the angular speed at $t = 5$.

53. Find the slope of the tangent line of the graph of the function $y = \dfrac{8a^3}{4a^2 + x^2}$ at the point with abscissa $x = 2a$.

Homework 4

54. Find y', if $x^4 + y^4 = x^2y^2$.

55. Find y', if $y \sin x - \cos(x - y) = 0$.

56. Find y', if $2y \ln y = x$.

57. Find y', if $2^x + 2^y = 2^{x+y}$.

58. Find y', if $y = x^{\frac{1}{x}}$.

59. Find y', if $y = \left(\dfrac{x}{1 + x}\right)^x$.

60. Find y', if $y = \dfrac{\sqrt{x - 2}}{(x + 3)^3 \sqrt[5]{x^2}}$.

61. Find $\dfrac{dy}{dx}$, if $x = t(1 - \sin t)$, $y = t \cos t$.

62. Find the slope of the tangent line of the ellipse $x = 2\cos t$, $y = \sin t$ at the point $A\left(1; -\dfrac{\sqrt{3}}{2}\right)$.

Homework 5

63. Express the differential dy of the function $y = xe^{2x}$

64. Evaluate the differential and the increment of the function $y = \ln \frac{x}{x^2+1}$, if $x = 2$ and $\Delta x = \frac{1}{30}$

65. Using the differential of function, evaluate the approximate value of $\ln 1,01$

66. Using the differential of function, evaluate the approximate value of $\sqrt[4]{16,64}$

67. Find y'', if $y = \sqrt{1+x^2}$

68. Find y'', if $y = x(\sin \ln x + \cos \ln x)$

69. Find $\frac{d^3y}{dx^3}$ for $y = \ln(1+x^2)$

70. Evaluate $f^{IV}(1)$ if $f(x) = x^6 - 4x^3 + 4$

71. Find $y^{(n)}$ of the function $y = \frac{x}{x+1}$

72. Find $\frac{d^ny}{dx^n}$ for $y = x \cdot 2^x$

Homework 6

In exercises 73. - 80. evaluate the limits using the L'Hospital's rule.

73. $\lim\limits_{x\to 2} \frac{\sqrt[3]{x} - \sqrt[3]{2}}{\sqrt{x} - \sqrt{2}}$

74. $\lim\limits_{x\to 0} \frac{e^{ax} - e^{bx}}{2x}$

75. $\lim\limits_{x\to \infty} \frac{\pi - 2\arctan x}{\ln\left(1 + \frac{1}{x}\right)}$

76. $\lim\limits_{x\to \infty} x^3 e^{-x}$

77. $\lim\limits_{x\to 1} \left(\frac{1}{\ln x} - \frac{x}{\ln x}\right)$

78. $\lim\limits_{x\to 0} x^{\sin x}$

79. $\lim\limits_{x\to 0} \left(\frac{1}{x}\right)^x$

80. $\lim\limits_{x\to 0} (e^x + x)^{\frac{1}{x}}$.

81. Using the Taylor's formula, expand the function $f(x) = x^5 - 3x^3 + 1$ in powers $x - 1$.

82. Compose the second order Taylor's formula of the function $y = \sin^2 x$ in the neighborhood of $x_0 = 0$. Using the polynomial obtained, evaluate the approximate value of $\sin^2 0,3$.

83. Compose the third order Taylor's formula of the function $y = x^3 \ln x$ in the neighborhood of $x_0 = 1$.

Homework 7

84. Find the intervals of increase and decrease of the function $y = \dfrac{x}{\ln x}$.

85. In given closed interval $[0; 2\pi]$ find the intervals in which the function $y = 2\sin x + \cos 2x$ is increasing and decreasing.

86. Find the local extrema of the function
$y = x - \ln(1 + x)$.

87. Find the local extrema of the function
$y = (x - 5)^2 \sqrt[3]{(x + 1)^2}$.

88. Find the local extrema of the function
$y = x \sin x + \cos x - \dfrac{1}{4}x^2$ in the closed interval $\left[-\dfrac{\pi}{2}; \dfrac{\pi}{2}\right]$.

89. Find the domains of convexity and concavity and the inflection points of the graph of the function $y = \dfrac{x^3}{x^2 + 3}$.

90. Find the domains of convexity and concavity and the inflection points of the graph of the function $y = e^{-x^2}$.

Homework 8

In exercises 91. - 114. find the indefinite integral

91. $\displaystyle\int \frac{\sqrt[3]{x^2} - \sqrt[4]{x}}{\sqrt{x}} dx$

92. $\displaystyle\int \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2 dx$

93. $\displaystyle\int e^x (1 + x^2 e^{-x}) dx$

94. $\displaystyle\int \cot^2 x dx$

95. $\displaystyle\int \frac{dx}{\sqrt{2 - 3x^2}}$

96. $\displaystyle\int \frac{(1 + 2x^2)dx}{x^2(1 + x^2)}$

97. $\displaystyle\int \frac{dx}{3x^2 + 1}$

98. $\int \frac{1+\cos^2 x}{1+\cos 2x} dx$

99. $\int \frac{dx}{x^2-5}$

Homework 9

100. $\int \sqrt{5-2x} dx$

101. $\int \frac{x^3 dx}{\sqrt{x^4+3}}$

102. $\int \tan x dx$

103. $\int \sin^4 x \cos x dx$

104. $\int \frac{e^x dx}{e^x+2}$

105. $\int \frac{dx}{x \ln x}$

106. $\int \frac{x dx}{x^4+1}$

107. $\int \frac{dx}{x\sqrt{1-\ln^2 x}}$

108. $\int \frac{1+x}{\sqrt{1-x^2}} dx$

109. $\int (x+2) \sin 2x dx$

110. $\int x 3^x dx$

111. $\int \ln(x^2+1) dx$

112. $\int \arccos x dx$

113. $\int \frac{2x+3}{3x+2} dx$

114. $\int \frac{x^3 dx}{x+1}$

Homework 10

In exercises 115. - 124. evaluate the definite integral

115. $\displaystyle\int_0^{16} \frac{dx}{\sqrt{x+9}-\sqrt{x}}$

116. $\displaystyle\int_1^{2} \frac{e^{\frac{1}{x}}dx}{x^2}$

117. $\displaystyle\int_1^{e^3} \frac{dx}{x\sqrt{1+\ln x}}$

118. $\displaystyle\int_0^{1} \frac{dx}{x^2+4x+5}$

119. $\displaystyle\int_1^{2} \frac{dx}{x+x^2}$

120. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^5 x \sin 2x dx$

121. $\displaystyle\int_0^{\pi} x^3 \sin x dx$

122. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{xdx}{\sin^2 x}$

123. $\displaystyle\int_0^{e-1} \ln(x+1)dx$

124. $\displaystyle\int_0^{1} \frac{\sqrt{x}dx}{1+x}$

Homework 11

125. Determine the area between the parabolas $y^2+8x=16$ and $y^2-24x=48$.

126. Determine the area of the region bounded by the astroid $x=a\cos^3 t$, $y=a\sin^3 t$.

127. Determine the area of the region bounded by the limaçon of Pascal $\varrho=2a(2+\cos\varphi)$.

128. Determine the length of the arc of the curve $y=\ln(1-x^2)$ between $x=0$ and $x=\dfrac{1}{2}$.

129. Determine the length of the curve $y=\sqrt{x-x^2}+\arcsin\sqrt{x}$.

130. Determine the length of the arc of the curve $x=a(\cos t+t\sin t)$, $y=a(\sin t-t\cos t)$ between $t=0$ and $t=\pi$.

131. Determine the length of the arc of the hyperbolic spiral $\varrho\varphi=1$ between $\varphi=\frac{3}{4}$ and $\varphi=\frac{4}{3}$.

Homework 12

1. Perform the calculations and express the result in the form $a+ib$.

$$a)\quad (3-2i)^2-(3+2i)^2, \qquad b)\quad (1+2i)^6, \qquad c)\quad (1+i+i^2+i^3)^{100},$$

$$d)\quad \frac{4-3i}{1+i}, \qquad e)\quad \frac{(1+2i)^2-(1-i)^3}{(3+2i)^3-(2+i)^2}, \qquad f)\quad \left(\frac{1-i}{i+1}\right)^8.$$

2. In each part solve for z:

$$a)\quad (i-z)+(2z-3i)=-2+7i, \qquad b)\quad (4-3i)\bar{z}=i.$$

3. In each part plot the point and sketch the vector that corresponds to the given complex number.

$$a)\quad 2+3i, \qquad b)\quad -3-2i, \qquad c)\quad -5i, \quad d)\qquad -2-2i$$

4. In each part express the complex number in polar form using its principal argument

$$a)\, 2i, \quad b)\, -4, \quad c)\, 5+5i, \quad d)\, -3-3i, \quad e)\, 2\sqrt{3}-2i, \quad f)\, -6-6\sqrt{3}i.$$

5. Given that $z_1 = 2(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$ and $z_2 = 3(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$, find the polar form of

$$a)\quad z_1z_2 \quad , \qquad b)\quad \frac{z_1}{z_2} \quad , \qquad c)\quad \frac{z_2^3}{z_1^2}$$

6. Express $z_1=i$, $z_2=1-i\sqrt{3}$ and $z_3=\sqrt{3}+i$ in polar form and use these results to find $\frac{z_1z_2}{z_3}$. Check your results by performing the calculations without using polar forms.

Homework 13

1. Express the given complex numbers in algebraic form, in polar form and as a point or a vector in a complex plane.

$$z_1 = \frac{1}{1-i\sqrt{3}}, \qquad z_2 = \frac{1}{2(cos\frac{\pi}{6}-isin\frac{\pi}{6})}$$

2. Find the modulus and the principal value of the argument of the complex number

$$z=(1-i)^4(3+3i)^2.$$

3. Calculate using polar forms

$$a)\quad \frac{(1+i)(1-\sqrt{3}i)^2}{(-3+3i)(2-2i)^5} \qquad b)\quad \left(\frac{-\sqrt{3}+i}{1-\sqrt{3}i}\right)^{40}$$

4. In each part find all the roots and sketch them as vectors in the complex plane.

$$a)\quad \sqrt[3]{-i} \qquad b)\quad \sqrt[6]{1} \qquad c)\quad \sqrt[4]{-8+8\sqrt{3}i}$$

5. Find

$$a)\quad \left(1-\frac{\sqrt{3}-i}{2}\right)^{24} \qquad b)\quad \sqrt[3]{\frac{2-i}{1+2i}}$$

$$c)\quad \sqrt[3]{\frac{(2\sqrt{2}+i2\sqrt{2})^2}{\sqrt{2}-i\sqrt{2}}} \qquad d)\quad \sqrt[4]{1+5i-\frac{6}{1-i}}$$

Homework 14

1. Let $A = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix}$. Find the following matrices (if possible):
$X = (-AC)^T + 5D^T$, $Y = A^T(2C - BA^T)^T$, $W = B^T(CC^T - A^TA)$.

2. Let $K = \begin{pmatrix} 1 & 2 \\ 3 & 0 \\ 5 & 6 \end{pmatrix}$ and $L = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & 0 \end{pmatrix}$. Find KL and LK.

3. Let $M = \begin{pmatrix} 1 & -3 & 2 \\ 3 & -4 & 1 \\ 2 & -5 & 3 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & 5 & 6 \\ 1 & 2 & 5 \\ 1 & 3 & 2 \end{pmatrix}$. Find $MN - NM$.

4. Find FGH, if

$$F = \begin{pmatrix} 991 & 992 & 993 \\ 994 & 995 & 996 \\ 997 & 998 & 999 \\ 1000 & 1001 & 1002 \end{pmatrix}, \quad G = \begin{pmatrix} 12 & -6 & -2 \\ 18 & -9 & -3 \\ 24 & -12 & -4 \end{pmatrix}, \quad H = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 3 & 0 \end{pmatrix}.$$

5. Let P and Q be the matrices of size 2×2.
a) Give an example in which $(P - Q)^2 \neq P^2 - 2PQ + Q^2$.
b) Fill in the blank to create a matrix identity that is valid for all choices of P and Q:

$$(P - Q)^2 = P^2 + Q^2 + \ldots\ldots\ldots\ldots\ldots$$

Homework 15

1. Solve the linear systems using Gaussian elimination

$$a)\ \begin{cases} x_1 + x_2 + 2x_3 = 8 \\ -x_1 - 2x_2 + 3x_3 = 1 \\ 3x_1 - 7x_2 + 4x_3 = 10 \end{cases} \qquad b)\ \begin{cases} 2x - y - 3z = 0 \\ -x + 2y - 3z = 0 \\ x + y + 4z = 0 \end{cases}$$

2. Reduce

$$\begin{pmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{pmatrix}$$

to reduced row-echelon form without introducing any fractions.

3. Solve the linear systems using Gaussian elimination and check your answer.

$$a)\ \begin{cases} 2x_1 - x_2 + 3x_3 + 4x_4 = 9 \\ x_1 - 2x_3 + 7x_4 = 11 \\ 3x_1 - 3x_2 + x_3 + 5x_4 = 8 \\ 2x_1 + x_2 + 4x_3 + 4x_4 = 10 \end{cases}, \qquad b)\ \begin{cases} 2x_1 - x_2 + x_3 + 2x_4 = 3 \\ 3x_1 + 2x_3 - x_4 = 7 \\ -x_1 + 2x_2 + x_4 = 1 \\ x_1 + 3x_2 + 4x_3 = 12 \end{cases},$$

$$c)\ \begin{cases} 9x_1 - 2x_2 + 2x_3 - 3x_4 = -15 \\ 11x_1 - 4x_2 + 3x_3 - 5x_4 = -26 \\ -18x_1 + 5x_2 - 4x_3 + 7x_4 = 35 \end{cases}, \qquad d)\ \begin{cases} 3x_1 + 2x_2 - 4x_3 + x_4 + x_5 = 11 \\ x_1 + x_2 - 2x_3 + 2x_4 - x_5 = 12 \\ x_1 + x_2 + 2x_3 - x_4 + x_5 = 8 \end{cases}$$

$$e)\quad \begin{cases} 2x+2y+4z &= 0 \\ w \qquad\quad -y-3z &= 0 \\ 2w+3x+y+z &= 0 \\ -2w+x+3y-2z &= 0 \end{cases}, \qquad f)\quad \begin{cases} x_1 \qquad\quad +10x_3-4x_4 &= 1 \\ x_1+\ x_2+4x_3-\ x_4 &= 2 \\ 2x_1+3x_2+2x_3+\ x_4 &= 5 \\ -2x_1-2x_2-8x_3+2x_4 &= -4 \\ x_2-6x_3+3x_4 &= 1 \end{cases}$$

Homework 16

1. Check the consistency of the system. If the system is consistent, then find the solution.

$$a)\quad \begin{cases} x+2y+3z+u &= 1 \\ 2x+3y+z+2u &= 4 \\ 3x+y+2z-2u &= 2 \\ 4y+2z+5u &= 3 \end{cases}, \qquad b)\quad \begin{cases} 2x_1-2x_2+x_3-x_4 &= 1 \\ x_1+2x_2-x_3+x_4 &= 1 \\ 4x_1-10x_2+5x_3-5x_4 &= 1 \\ 2x_1-14x_2+7x_3-7x_4 &= -1 \end{cases}$$

$$c)\quad \begin{cases} x_1-2x_2+x_3-4x_4 = 1 \\ x_1+3x_2+7x_3+2x_4 = 2 \\ x_1-12x_2-11x_3-16x_4 = 5 \end{cases}$$

2. For which values of λ does the system of equations

$$\begin{cases} (\lambda-3)x+ \qquad y &= 0 \\ x+(\lambda-3)y &= 0 \end{cases}$$

have nontrivial solutions?

3. For which values of a will the following system have no solutions? Exactly one solution? Infinitely many solutions?

$$\begin{cases} x+2y \qquad -3z &= 4 \\ 3x-y \qquad +5z &= 2 \\ 4x+y+(a^2-14)z &= a+2 \end{cases}$$

4. How does the existence and number of solutions depend on parameters a and b.

$$\begin{cases} x+ay+4z &= 4 \\ 5x+y+2z &= 3 \\ 3x-y+z &= b \end{cases}$$

Homework 17

1. Find the inverses of the given matrices and check the result

$$\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & 1 & 0 \\ -1 & 2 & 1 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}, \quad \begin{pmatrix} 3 & 2 & 1 & 2 \\ 7 & 5 & 2 & 5 \\ 0 & 0 & 9 & 4 \\ 0 & 0 & 11 & 5 \end{pmatrix},$$

2. Solve the matrix equation (find the matrix X).

$$a)\quad \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} X \begin{pmatrix} 2 & -2 \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$$

$$b)\quad \begin{pmatrix} 2 & -3 & 1 \\ 4 & -5 & 2 \\ 5 & -7 & 3 \end{pmatrix} X \begin{pmatrix} 9 & 7 & 6 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -2 \\ 18 & 12 & 9 \\ 23 & 15 & 11 \end{pmatrix}$$

$$c)\quad \begin{pmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{pmatrix} X = \begin{pmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{pmatrix}$$

$$d)\quad X \begin{pmatrix} -1 & 0 & 1 \\ 1 & 1 & 0 \\ 3 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 \\ -3 & 1 & 5 \end{pmatrix}$$

Homework 18

Find all solutions of the differential equation:

$$1.\quad y'y = 1.$$

$$2.\quad 2x\sqrt{1-y^2} + yy' = 0.$$

$$3.\quad 2x^2yy' + y^2 = 2.$$

Find the solution of the differential equation using the additional equation:

$$4.\quad dy\sin x - y\cos x dx = 0, \qquad y(\frac{\pi}{2}) = 1.$$

$$5.\quad \sin x \sin y dx + \cos x \cos y dy = 0, \qquad y(\frac{\pi}{4}) = \frac{\pi}{4}.$$

Solve the differential equations using appropriate change of variables:

$$6.\quad y' = (x+y)^2$$

$$7.\quad y' = \sqrt{y-x} + 1$$

Homework 19

Solve the differential equations:

1. $y' + y\cos x = e^{-\sin x}$
2. $xdy + (x^2 - y)dx = 0$
3. $y = x(y' - x\cos x)$
4. $x^2y' + (1-2x)y - x^2 = 0$
5. $y'' - 7y' + 12y = 0$
6. $y'' + 6y' + 9y = 0$
7. $y'' + 2y' + 5y = 0$

Homework 20

Solve the linear differential equations:

1. $y'' - 7y' + 12y = 12x$
2. $4y'' + 8y' = \sin x$
3. $y'' - 6y' + 9y = 25e^x \sin x$
4. $y'' + y' = 3$
5. $y'' + 8y' = 8x$
6. $y'' - y = 4e^x$
7. $y''' - y'' = 12x^2 + 6x$
8. $y'' - 3y' + 2y = 2$, if in addition $y(0) = 2$ and $y'(0) = 1$.
9. $y'' - 5y' + 6y = e^{-x}$, if it is known that in addition $y(0) = 0$ and $y'(0) = 0$.

Homework 21

1. Evaluate the determinants

$$\begin{vmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix}, \quad \begin{vmatrix} 0 & 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ -\frac{1}{3} & \frac{2}{3} & 0 & 0 \end{vmatrix}, \quad \begin{vmatrix} 6 & 3 & 2 & 3 \\ 4 & 2 & 1 & 2 \\ 2 & -1 & -3 & 1 \\ 1 & -3 & 4 & -3 \end{vmatrix},$$

$$\begin{vmatrix} 2 & -3 & 7 & 1 & 9 & 11 \\ 1 & 0 & 3 & 0 & -4 & 0 \\ 7 & 4 & 9 & -1 & 11 & -5 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 9 & -4 & 11 & 1 & 13 & 2 \\ 4 & 0 & 1 & 0 & -1 & 0 \end{vmatrix}, \quad \begin{vmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 & 1 & 2 \\ 4 & 3 & 2 & 1 & 0 & 1 \\ 5 & 4 & 3 & 2 & 1 & 0 \end{vmatrix}$$

2. Find $det(AB)$, if $A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \\ 5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & 0 \end{pmatrix}$
3. Verify, that the equality $det(CD) = det(C)det(D)$ holds for the matrices

$$C = \begin{pmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 5 & 0 & 6 \end{pmatrix} \quad .$$

Homework 22

1. Find the eigenvalues and eigenvectors of the following matrices

$$a) \quad \begin{pmatrix} 4 & -2 & -3 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix},$$

$$b) \quad \begin{pmatrix} 1 & 3 & 0 \\ 3 & -2 & -1 \\ 0 & -1 & 1 \end{pmatrix}.$$

Answers

1. $X=(-\infty;-5)\cup(-5;0)$; 2. $X=[-1;3]$; 3. $X=[-4;-\pi]\cup[0;\pi]$; 4: $X=(4;5)\cup(6;\infty)$; 5. $Y=[-1;3]$ 6. $Y=[0;2]$; 7. $y=2\pm\sqrt{1+x}$; 8. $y=\log_2\frac{x}{1-x}$; 9. $y=\ln\left(10^{1-x}-3\right)$; 10. $y=\pm\cos\frac{x}{4}$ $(0\le x\le 2\pi$; 11. $y=8x-8$; 12. $y=\arccos\frac{x^2}{1+x}$; 14. odd; 15. even; 16. neither; 17. even; 18. $-\frac{1}{x}$; 19. 0; 20. $\frac{5}{6}$; 21. 4; 21. 6; 22. $3\frac{3}{7}$; 23. ∞; 24. 1; 25. $-\frac{1}{56}$; 26. $\frac{1}{2\sqrt{x}}$; 27. 2; 28. $\frac{1}{2}$; 29. $-\frac{1}{2}$; 30. ∞; 31. 0;32. $\frac{2}{5}$; 33. $\frac{3}{4}$; 34. 0; 35. 1; 36. e^2; 37. e^3; 38. $\frac{1}{e}$; 39. e^2; 42. $\frac{x-2}{x^3}$; 43. $\frac{2x+2}{(x^2+2x+4)\ln 3}$; 44. $10^{\sqrt{x}}\left(1+\frac{\sqrt{x}\ln 10}{2}\right)$; 45. $\frac{1-x}{\sqrt{1+x^2}}$; 46. $\frac{54\sqrt[5]{x^4}}{55\cdot\sqrt[11]{\left(9+6\sqrt[5]{x^9}\right)^{10}}}$; 47. $\frac{(e^x+e^{-x})(\cos x-\sin x)}{e^x\cos x+e^{-x}\sin x}$; 48. $\frac{x^2}{\sqrt{1-2x-x^2}}$; 49. $\frac{x^5+1}{x^4(1+x^2)}$; 50. $-\cos 2x$; 51. 1; 52. $13\frac{rad}{s}$; 53. $-\frac{1}{2}$; 54 $\frac{x}{y}\cdot\frac{y^2-2x^2}{2y^2-x^2}$; 55. $\frac{y\cos x+\sin(x-y)}{\sin(x-y)-\sin x}$; 56. $\frac{1}{2(1+\ln y)}$; 57. $2^{x-y}\cdot\frac{2^y-1}{1-2^x}$; 58. $x^{\frac{1}{x}-2}(1-\ln x)$;

59. $\left(\frac{x}{1+x}\right)^x\left(\frac{1}{1+x}+\ln\frac{x}{1+x}\right)$; 60. $\frac{\sqrt{x-2}}{(x+3)^3\cdot\sqrt[5]{x^2}}\left[\frac{1}{2(x-2)}-\frac{3}{x+3}-\frac{2}{5x}\right]$;

61. $\frac{\cos t-t\sin t}{1-\sin t-t\cos t}$; 62. $\frac{\sqrt{3}}{6}$; 63. $e^{2x}(1+2x)dx$; 64. $dy=-0,01$, $\Delta y=-0,0100044$; 65. 0,01; 66. 2,02; 67. $\frac{1}{(1+x^2)\sqrt{1+x^2}}$; 68. $-\frac{2\sin\ln x}{x}$; 69. $\frac{4x(x^2-3)}{(1+x^2)^3}$ 70. 360; 71. $(-1)^{n+1}\cdot\frac{n!}{(x+1)^{n+1}}$; 72. $2^x\ln^{n-1}2\cdot(n+x\ln 2)$; 73. $\frac{2}{3\cdot\sqrt[6]{2}}$; 74. $\frac{a-b}{2}$; 75. 2; 76. 0; 77. -1; 78. 1; 79. 1; 80. e^2; 81. $-1-4(x-1)+(x-1)^2+7(x-1)^3+5(x-1)^4+(x-1)^5$; 82. $x^2+R_2(x)$, where $R_2(x)=-\frac{2x^3\sin 2\Theta x}{3}$; 83. $x-1+\frac{5}{2}(x-1)^2+\frac{11}{6}(x-1)^3+R_3(x)$, where $R_3(x)=\frac{(x-1)^4}{4[1+\Theta(x-1)]}$; 84. $X\uparrow=(e;\infty)$, $X\downarrow=(0;1)$, $X\downarrow=(1;e)$; 85. $X\uparrow=\left(0;\frac{\pi}{6}\right)$, $X\uparrow=\left(\frac{\pi}{2};\frac{5\pi}{6}\right)$, $X\uparrow=\left(\frac{3\pi}{2};2\pi\right)$, $X\downarrow=\left(\frac{\pi}{6};\frac{\pi}{2}\right)$, $X\downarrow=\left(\frac{5\pi}{6};\frac{3\pi}{2}\right)$; 86. At $x=0$ local minimum 87. At $x=-1$ and $x=5$ local minima, at $x=0,5$ local maximum 88. At $x=0$ local minimum. At $x=\pm\frac{\pi}{3}$ local maximum 89. Convex in intervals $(-3;0)$ and $(3;\infty)$, concave in intervals $(-\infty;-3)$ and $(0;3)$, inflection points $(-3;-\frac{9}{4})$, $(0;0)$ and $(3;\frac{9}{4})$; 90. Convex in interval $(-\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}})$, concave in intervals $(-\infty;-\frac{1}{\sqrt{2}})$ and $(\frac{1}{\sqrt{2}};\infty)$, inflection points $(-\frac{1}{\sqrt{2}};\frac{1}{\sqrt{e}})$ and $(\frac{1}{\sqrt{2}};\frac{1}{\sqrt{e}})$; 91. $\frac{6x\sqrt[6]{x}}{7}-\frac{4\sqrt[4]{x^3}}{3}+C$; 92. $x-\cos x+C$; 93. $e^x+\frac{x^3}{3}+C$; 94. $-\cot x-x+C$; 95. $\frac{1}{\sqrt{3}}\arcsin\frac{x\sqrt{3}}{\sqrt{2}}+C$; 96. $\arctan x-\frac{1}{x}+C$; 97. $\frac{\sqrt{3}}{3}\arctan(x\sqrt{3})+C$; 98.

$\frac{1}{2}(\tan x + x) + C$; 99. $\frac{1}{2\sqrt{5}} \ln\left|\frac{\sqrt{5}-x}{\sqrt{5}+x}\right| + C$; 100. $\frac{(2x-5)\sqrt{5-2x}}{3} + C$; 101. $\frac{1}{2}\sqrt{x^4+3} + C$; 102. $-\ln|\cos x| + C$; 103. $\frac{\sin^5 x}{5} + C$; 104. $\ln(e^x+2) + C$; 105. $\ln|\ln x| + C$; 106. $\frac{1}{2}\arctan x^2 + C$; 107. $\arcsin \ln x + C$; 108. $\arcsin x - \sqrt{1-x^2} + C$; 109. $-\frac{(x+2)\cos 2x}{2} + \frac{\sin 2x}{4} + C$; 110. $\frac{x3^x}{\ln 3} - \frac{3^x}{\ln^2 3} + C$; 111. $x\ln(x^2+1) - 2x + 2\arctan x + C$; 112. $x \arccos x - \sqrt{1-x^2} + C$; 113. $\frac{2x}{3} + \frac{5}{9}\ln|3x+2| + C$; 114. $\frac{x^3}{3} - \frac{x^2}{2} + x - \ln|x+1| + C$; 115. 12; 116. $e - \sqrt{e}$; 117. 2; 118. $\arctan 3 - \arctan 2$; 119. $\ln\frac{4}{3}$; 120. $\frac{2}{7}$; 121. $\pi^3 - 6\pi$; 122. $\frac{\pi(9-4\sqrt{3})}{36} + \frac{1}{2}\ln\frac{3}{2}$; 123. 1; 124. $2 - \frac{\pi}{2}$; 125. $\frac{32}{3}\sqrt{6}$; 126. $\frac{3}{8}\pi a^2$; 127. $18\pi a^2$; 128. $\ln 3 - \frac{1}{2}$; 129. 2; 130. $\frac{\pi^2 a}{2}$; 131. $\ln\frac{3}{2} + \frac{5}{12}$;