Numbrilise diferentseerimise valemid

Järgnevad valemid kehtivad ühtlase võrgu jaoks, st $x_i = x_0 + ih$, $i = 1, 2, \ldots$. Tähistame $y_i = f(x_i)$, $i = 0, 1, \ldots$ ning ξ, ξ_1 ja ξ_2 tähistavad suurusi valemis kasutatavate sõlmede vahel. Valemid koos jääkliikmetega kehtivad, kui jääkliikmes esinevad tuletised on sõlmi sisaldaval lõigul pidevad.

Esimest järku tuletised

$n = 1$, st vaadeldakse kahte sõlme

$$f'(x_0) = \frac{1}{h}(y_1 - y_0) - \frac{h}{2}f''(\xi)$$

$$f'(x_1) = \frac{1}{h}(y_1 - y_0) + \frac{h}{2}f''(\xi)$$

$n = 2$, st vaadeldakse kolme sõlme

$$f'(x_0) = \frac{1}{2h}(-3y_0 + 4y_1 - y_2) + \frac{h^2}{3}f'''(\xi)$$

$$f'(x_1) = \frac{1}{2h}(y_2 - y_0) - \frac{h^2}{6}f'''(\xi)$$

$$f'(x_2) = \frac{1}{2h}(y_0 - 4y_1 + 3y_2) + \frac{h^2}{3}f'''(\xi)$$

$n = 3$, st vaadeldakse nelja sõlme

$$f'(x_0) = \frac{1}{6h}(-11y_0 + 18y_1 - 9y_2 + 2y_3) - \frac{h^3}{4}f^{IV}(\xi)$$

$$f'(x_1) = \frac{1}{6h}(-2y_0 - 3y_1 + 6y_2 - y_3) + \frac{h^3}{12}f^{IV}(\xi)$$

$$f'(x_2) = \frac{1}{6h}(y_0 - 6y_1 + 3y_2 + 2y_3) - \frac{h^3}{12}f^{IV}(\xi)$$

$$f'(x_3) = \frac{1}{6h}(-2y_0 + 9y_1 - 18y_2 + 11y_3) + \frac{h^3}{4}f^{IV}(\xi)$$

$n = 4$, st vaadeldakse viit sõlme

$$f'(x_0) = \frac{1}{12h}(-25y_0 + 48y_1 - 36y_2 + 16y_3 - 3y_4) + \frac{h^4}{5}f^{V}(\xi)$$

$$f'(x_1) = \frac{1}{12h}(-3y_0 - 10y_1 + 18y_2 - 6y_3 + y_4) - \frac{h^4}{20}f^{V}(\xi)$$

$$f'(x_2) = \frac{1}{12h}(y_0 - 8y_1 + 8y_3 - y_4) + \frac{h^4}{30}f^{V}(\xi)$$

$$f'(x_3) = \frac{1}{12h}(-y_0 + 6y_1 - 18y_2 + 10y_3 + 3y_4) - \frac{h^4}{20}f^{V}(\xi)$$

$$f'(x_4) = \frac{1}{12h}(3y_0 - 16y_1 + 36y_2 - 48y_3 + 25y_4) + \frac{h^4}{5}f^{V}(\xi)$$

Kõrgemat järku tuletised

$n = 2$ st vaadeldakse kolme sõlme

$$f''(x_0) = \frac{1}{h^2}(y_0 - 2y_1 + y_2) - hf'''(\xi_1) + \frac{h^2}{6}f^{IV}(\xi_2)$$

$$f''(x_1) = \frac{1}{h^2}(y_0 - 2y_1 + y_2) - \frac{h^2}{12}f^{IV}(\xi)$$

$$f''(x_2) = \frac{1}{h^2}(y_0 - 2y_1 + y_2) + hf'''(\xi_1) + \frac{h^2}{6}f^{IV}(\xi_2)$$

$n = 3$ st vaadeldakse nelja sõlme

$$f''(x_0) = \frac{1}{h^2}(2y_0 - 5y_1 + 4y_2 - y_3) + \frac{11}{12}h^2 f^{IV}(\xi_1) - \frac{h^3}{10}f^{V}(\xi_2)$$

$$f''(x_1) = \frac{1}{h^2}(y_0 - 2y_1 + y_2) - \frac{1}{12}h^2 f^{IV}(\xi_1) + \frac{h^3}{30}f^{V}(\xi_2)$$

$$f''(x_2) = \frac{1}{h^2}(y_1 - 2y_2 + y_3) - \frac{1}{12}h^2 f^{IV}(\xi_1) - \frac{h^3}{30}f^{V}(\xi_2)$$

$$f''(x_3) = \frac{1}{h^2}(-y_0 + 4y_1 - 5y_2 + 2y_3) + \frac{11}{12}h^2 f^{IV}(\xi_1) + \frac{h^3}{10}f^{V}(\xi_2)$$

$n = 4$ st vaadeldakse viit sõlme

$$f''(x_0) = \frac{1}{12h^2}(35y_0 - 104y_1 + 114y_2 - 56y_3 + 11y_4) - \frac{5}{6}h^3 f^{V}(\xi_1) + \frac{h^4}{15}f^{VI}(\xi_2)$$

$$f''(x_1) = \frac{1}{12h^2}(11y_0 - 20y_1 + 6y_2 + 4y_3 - y_4) + \frac{1}{12}h^3 f^{V}(\xi_1) - \frac{h^4}{60}f^{VI}(\xi_2)$$

$$f''(x_2) = \frac{1}{12h^2}(-y_0 + 16y_1 - 30y_2 + 16y_3 - y_4) + \frac{1}{90}h^4 f^{VI}(\xi)$$

$$f''(x_3) = \frac{1}{12h^2}(-y_0 + 4y_1 + 6y_2 - 20y_3 + 11y_4) - \frac{1}{12}h^3 f^{V}(\xi_1) - \frac{h^4}{60}f^{VI}(\xi_2)$$

$$f''(x_4) = \frac{1}{12h^2}(11y_0 - 56y_1 + 114y_2 - 104y_3 + 35y_4) + \frac{5}{6}h^3 f^{V}(\xi_1) + \frac{h^4}{15}f^{VI}(\xi_2)$$

$$f''(x_3) = \frac{1}{180h^2}(2y_0 - 27y_1 + 270y_2 - 490y_3 + 270y_4 - 27y_5 + 2y_6) - \frac{1}{560}h^6 f^{(8)}(\xi)$$

$$f'''(x_2) = \frac{1}{2h^3}(-y_0 + 2y_1 - 2y_3 + y_4) - \frac{1}{4}h^2 f^{(5)}(\xi)$$

$$f'''(x_3) = \frac{1}{8h^3}(y_0 - 8y_1 + 13y_2 - 13y_4 + 8y_5 - y_6) - \frac{7}{120}h^4 f^{(7)}(\xi)$$

$$f^{IV}(x_2) = \frac{1}{h^4}(y_0 - 4y_1 + 6y_2 - 4y_3 + y_4) - \frac{1}{6}h^2 f^{(6)}(\xi)$$

$$f^{IV}(x_3) = \frac{1}{6h^4}(-y_0 + 12y_1 - 39y_2 + 56y_3 - 39y_4 + 12y_5 - y_6) + \frac{7}{240}h^4 f^{(8)}(\xi)$$