Questions for tests on theory

After questions, references to appropriate pages in the textbook of P. Drjbek and G. Holubovj are given in brackets. But learning the material by means of this textbook is not obligatory. You can use other sources as well (e.g. slides of lecturer).

At a test, 1 longer question and 2 - 3 shorter questions will be given. During preparation of answer to the longer question, any auxiliary material (e.g. the textbook, slides of lecturer) may be used. During preparation of answers to shorter questions, usage of auxiliary materials is not allowed. The time for preparation of answer to the longer question is max 60 min and the time for preparation of answers to the shorter questions is max 30 min.

Questions for the first test on theory

Chapter 1

Longer questions

1. Derive the evolution conservation law in three-dimensional case. (p. 1-4)
2. Derive the evolution conservation law in one-dimensional case. (p. 5-6)
3. Derive the transport equation and one-dimensional diffusion equation from the conservation law making use of suitable constitutive laws. (p. 7-10)
4. Derive the equation of vibrating string. (p. 11-14)

Shorter questions

1. Write the diffusion equation in one and multiple dimensions (p. 9, 11)
2. Write the wave equation in one and multiple dimensions (p. 14, 16)
3. Write the Poisson and Laplace equation (p. 16, 17)
4. Describe one physical phenomenon governed by the Poisson equation. (p. 16-17)
5. What is the harmonic function? (p. 16)

Chapter 2

Longer questions

1. Present the Dirichlet, Neumann and Robin boundary conditions and describe their meaning in case of some physical models. When a boundary condition is homogeneous, nonhomogeneous? (p. 26-28)

2. Present the classification of linear partial differential equations of the second order. (p. 30, 32)

Shorter questions

1. What is the linear partial differential equation, when this equation is homogeneous and when it is nonhomogeneous? (p. 21, 22)

2. What is the classical solution of a partial differential equation? (p. 23)

3. What is the general solution of a partial differential equation? (p. 25)

4. Formulate one initial boundary value problem for an evolution equation. (p. 28)

5. Which problem is well-posed and which one is ill-posed? (p. 28-29)

Chapter 3

Longer questions

1. Derive the general solution of the equation

$$a(x,y)u_x(x,y) + b(x,y)u_y(x,y) = 0$$

and reduce the equation

$$a(x,y)u_x(x,y) + b(x,y)u_y(x,y) + c(x,y)u(x,y) = f(x,y)$$

to an ordinary differential equaton of by means of the method of characteristic coordinates. (p. 46-47, 48-49)

2. Formulate an existence and uniqueness theorem for the solution of the equation

$$a(x,y)u_x(x,y) + b(x,y)u_y(x,y) + c(x,y)u(x,y) = f(x,y)$$

with the side condition given on a parametric curve

$$\gamma \quad : \quad x = x_0(s), \; y = y_0(s).$$

Why the curve γ must intersect the characteristics transversally? (p. 51)

Shorter questions

1. What is called a characteristic of a linear partial differential equation of the first order and what is its meaning? (p. 38)

Chapter 4

Longer questions

1. Derive the solution formula of the problem

$$u_{tt}(x,t) - c^2 u_{xx}(x,t) = 0, \quad x \in \mathbb{R},\ t > 0$$
$$u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x)\,, \quad x \in \mathbb{R}$$

   (i.e. d'Alemberts's formula) (p. 67)

2. Derive the solution formula of the problem

$$u_{tt}(x,t) - c^2 u_{xx}(x,t) = f(x,t), \quad x \in \mathbb{R},\ t > 0$$
$$u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x)\,, \quad x \in \mathbb{R}$$

   either by means of the Green's theorem or by means of the operator method. (p. 76-79)

Shorter questions

1. What is the domain of influence of a point $(x_0, 0)$? (p. 73)
2. What is the domain of dependence of a point (x, t)? (p. 73-74)

Chapter 5

Longer questions

1. Derive the solution formula of the problem

$$u_t(x,t) - k u_{xx}(x,t) = 0, \quad x \in \mathbb{R},\ t > 0$$
$$u(x,0) = \varphi(x)\,, \quad x \in \mathbb{R}$$

   (p. 84-88)

2. Derive the solution formula of the problem

$$u_t(x,t) - ku_{xx}(x,t) = f(x,t), \quad x \in \mathbb{R},\ t > 0$$
$$u(x,0) = \varphi(x)\,, \quad x \in \mathbb{R}$$

by means of the operator method. (p. 91-93)

Questions for the second test on theory

Chapter 6

Longer questions

1. Solve the Poisson equation

$$\Delta u(\mathbf{x}) = f(\mathbf{x})$$

in the radially symmetric case (p. 100)

Shorter questions

1. For which transformations the Laplace operator is invariant? (p. 97)

Chapter 7

Longer questions

1. Derive the solution formula for the problem

$$u_t(x,t) - ku_{xx}(x,t) = 0\,, \quad x > 0,\ t > 0$$
$$u(0,t) = 0\,, \quad t > 0$$
$$u(x,0) = \varphi(x)\,, \quad x > 0,$$

by means of the reflection method. (p. 103–104)

2. Derive the solution formula for the problem

$$u_{tt}(x,t) - c^2u_{xx}(x,t) = 0\,, \quad 0 < x < l,\ t > 0$$
$$u(0,t) = u(l,t) = 0, \quad t > 0$$
$$u(x,0) = \varphi(x)\,, \quad u(x,0) = \psi(x)\,, \quad 0 < x < l.$$

(p. 111–113)

3. Derive the solution formula for the problem

$$u_t(x,t) - ku_{xx}(x,t) = 0\,,\ \ 0 < x < l,\ t > 0$$
$$u(0,t) = u(l,t) = 0,\ \ t > 0$$
$$u(x,0) = \varphi(x)\,,\ \ 0 < x < l.$$

(p. 116–117)

3. Solve the problem

$$u_t(x,t) - ku_{xx}(x,t) = f(x,t)\,,\ \ 0 < x < l,\ t > 0$$
$$u(0,t) = u(l,t) = 0,\ \ t > 0$$
$$u(x,0) = \varphi(x)\,,\ \ 0 < x < l.$$

by means of the Fourier method (p. 126–127)

Shorter questions

1. Which type of extension (even or odd) of initial conditions is used to solve initial boundary value problems for homogeneous diffusion and wave equations on the half-line $x > 0$:
a) in case of the Dirichlet' boundary condition;
b) in case of the Neumann boundary condition? (p. 103, 105, 107)

2. Which type of Fourier series (Fourier sine series or Fourier cosine series) is used to represent solutions of initial boundary value problems for homogeneous diffusion and wave equations on the finite interval $0 < x < l$:
a) in case of the Dirichlet' boundary condition;
b) in case of the Neumann boundary condition? (p. 113, 117, 118)

3. How the case of nonhomogeneous boundary conditions can be handled? (p. 109, 127–129)

Chapter 8 is skipped.

Chapter 9

Longer questions

1. Define the Laplace and Fourier transform and present their main properties. (p. 150–151, 153, 156–158)

Chapter 10 is skipped.

Chapter 11

Longer questions

1. Derive the Green's first identity and use it to prove the uniqueness of solution of the Dirichlet problem for Poisson equation. (p. 190, 193–194)
2. Derive the Green's second identity and the representation formula for harmonic functions in three-dimensional case. (p. 195–196)
3. Define the Green's function for Laplace operator corresponding to Dirichlet boundary conditions and derive the solution formula for the Laplace equation with Dirichlet boundary conditions in three-dimensional case. (p. 197–198)

Shorter questions

1. How many solutions has Neumann problem for Poisson equation? (p. 194)

Chapters 12,13

Longer questions

1. Derive the solution formula for the Cauchy problem for homogeneous diffusion equation in three-dimensional case

$$\begin{aligned} &u_t(\mathbf{x},t) - k\Delta u(\mathbf{x},t) = 0, \quad \mathbf{x} \in \mathbb{R}^3\,,\ t > 0 \\ &u(\mathbf{x},0) = \varphi(\mathbf{x}) \end{aligned}$$

   (p. 209–211)

2. Describe the Fourier method to solve the boundary value problem for diffusion equation

$$\begin{aligned} &u_t(\mathbf{x},t) - k\Delta u(\mathbf{x},t) = 0, \quad \mathbf{x} \in \Omega\,,\ t > 0 \\ &u(\mathbf{x},t) = 0, \quad \mathbf{x} \in \Gamma_1 \\ &\tfrac{\partial}{\partial \mathbf{n}} u(\mathbf{x},t) = 0, \quad \mathbf{x} \in \Gamma_2 \\ &\tfrac{\partial}{\partial \mathbf{n}} u(\mathbf{x},t) + au(\mathbf{x},t) = 0, \quad \mathbf{x} \in \Gamma_3 \\ &u(\mathbf{x},0) = \varphi(\mathbf{x}), \end{aligned}$$

   where $\Omega \subset \mathbb{R}^N$, $N \in \{2;3\}$, $\Gamma_1 \cup \Gamma_2 \cup \Gamma_3 = \partial\Omega$. (p. 213–214)

3. Present the Kirchhoff's formula for the solution of the Cauchy problem for homogeneous wave equation in three-dimensional case

$$u_{tt}(\mathbf{x},t) - c^2 \Delta u(\mathbf{x},t) = 0, \quad \mathbf{x} \in \mathbb{R}^3\,,\ t > 0$$
$$u(\mathbf{x},0) = \varphi(\mathbf{x}), \quad u_t(\mathbf{x},0) = \psi(\mathbf{x})$$

(p. 225)

Shorter questions

1. What is the Huygens' principle (p. 228)