SAMPLE

1.(a)[10 points] Determine the order of equation and whether it is nonlinear, linear nonhomogeneous, or linear homogeneous. Explain your reasoning.(more: Ex.2 p.33)

$$u_{xxx} + xy^3 u_{yy} = y + \ln x.$$

1.(b) [10 points] Determine in which regions of the xy-plane the equation is elliptic, hyperbolic, or parabolic.(more: Ex.10 p.33)

$$\sin x u_{xx} - 2 \cos x u_{xy} + \sin x u_{yy} + 5u_x - x^3 y^5 = 0.$$

2. [20 points] Find general solutions of the following homogeneous equation.(more: Ex.1-5, 8, 17 p.60)

$$16u_x - 7u_y = 0.$$

3. [20 points] Solve the Cauchy problem. (more: Ex.7, 13, 14, 16 p.60)

$$u_t + 5u_x = 4u + 8, \quad x \in \mathbb{R}, \, t > 0,$$
$$u(x, 0) = \arctan x + 5, \quad x \in \mathbb{R}.$$

4. [20 points] Solve the Cauchy problem for the wave equation. (more :Ex.4-8, 14-18 p.79)

$$u_{tt} - 9u_{xx} = t \cos 5x, \quad x \in \mathbb{R}, \, t > 0,$$
$$u(x, 0) = \ln(x^5 + 3), \quad x \in \mathbb{R},$$
$$u_t(x, 0) = xe^{7x}, \quad x \in \mathbb{R}.$$

5. [20 points] Solve the Cauchy problem for the diffusion equation. (more :Ex 6-9 p.95)

$$u_t = 4u_{xx}, \quad x \in \mathbb{R}, \, t > 0,$$
$$u(x, 0) = \begin{cases} 1 \text{ for } x > 0, \\ e^{2x} \text{ for } x < 0. \end{cases}$$