

The course content including exercises and examples can be found in the book *P. Drbek, G. Holubov, Elements of Partial Differential Equations, De Gruyter 2014*. The referece to the book is on the course website.

06.02.18. Classification, Types of Equations:

(Exercises after Ch.2, p. 32)

Class: Ex.1 (a, b, d, e), Ex.2 (a, b, c, h), Ex.10 (a, b, c, d, e), Ex.14 (a, b), Ex.7.

Home: Ex.2 (d, e, f, g, i), Ex.10 (f – parabolic, g – hyperbolic, h – elliptic, i – hyperbolic), Ex.14 (d – without type), Ex.5, Ex.6.

13.02.18. Linear PDEs of the First Order with Constant Coefficients.

Method of Characteristics, Coordinate Method:

Class: Linear ODEs of the first order

$$(a)y' = \frac{y}{x} - 1, \quad (b)y' + 3y = e^{-2x}, \quad (c)y' - 2y = xe^{2x}, \quad (d)y' - y = \sin 3x.$$

(Exercises after Ch.3, p. 60) Ex. 1, Ex. 3, Ex. 7.

Home:

$$(a)y' + \frac{2y}{x} = x^3 \quad \left[y = \frac{1}{6}x^4 + \frac{c}{x^2} \right], \quad (b)y' + 5y = e^x \quad \left[y = ce^{-5x} + \frac{1}{6}e^x \right],$$
$$(c)y' - 7y = e^{7x} \quad \left[y = ce^{7x} + xe^{7x} \right], \quad (d)y' + y = \cos 2x \quad \left[y = ce^{-x} + \frac{1}{5} \cos 2x + \frac{2}{5} \sin 2x \right].$$

(Exercises after Ch.3, p. 60) Ex. 2, Ex. 6, Ex. 11.

20.02.18. Linear PDEs of the First Order. Method of Characteristic Coordinates. Method of Characteristics:

(Exercises after Ch.3, p. 60)

Class: Ex.8, Ex. 4, Ex. 17(a, b, e), Ex. 18, Ex. 19.

Home: Ex.5, Ex. 9, Ex. 12, Ex. 17(c) [$u(x, y) = f(x/y)$], Ex. 17(d) [$u(x, y) = f(y + 0.5e^{-x^2})$], Ex. 13, Ex. 16, Ex. 26.

27.02.18. Wave equations in one spatial variable:

(Exercises after Ch.4, p. 79)

Class: Ex. 3(b), Ex. 4, Ex. 7, Ex. 6.

Home: Ex.3(a), Ex. 5, Ex. 8, Ex. 11.

06.03.18. Wave equations in one spatial variable. Cauchy problem for the nonhomogeneous equation:

(Exercises after Ch.4, p. 79)

Class: Ex. 13, Ex. 15, Ex. 16, Ex. 19.

Home: Ex. 14, Ex. 17, Ex. 18.

13.03.18. Diffusion equations in one spatial variable. Cauchy problem:

(Exercises after Ch.5, p. 94)

Class: Ex. 6, Ex. 8, Ex. 11, Ex. 12.

Home: Ex. 7, Ex. 10, Ex. 14.

27.03.18. Laplace and Poisson equations in 2D. Initial boundary value problems on half-line:

Class: Ex. 1(d), Ex. 2(b), Ex. 12, Ex. 13 p.101-102; Ex. 5, Ex. 7 p.131-132.

Home: Ex. 2(a, c, d, e) p.101; Ex. 4, Ex. 6 p.131-132.

03.04.18. Initial boundary value problems on finite interval. Fourier method:

(Exercises after Ch.7, p. 131)

Class: Ex. 12(d, i), Ex. 24(b).

Home: Ex. 12(c, e), Ex. 22(b, c).

10.04.18. Initial boundary value problems on finite interval. Fourier method for nonhomogeneous equations:

(Exercises after Ch.7, p. 131)

Class: Ex. 24(b).

$$u_{tt} = u_{xx} + Ae^{-t} \cos\left(\frac{\pi}{2l}x\right), \quad 0 < x < l, t > 0,$$

$$u_x(0, t) = u(l, t) = 0, \quad t > 0,$$

$$u(x, 0) = u_t(x, 0) = 0, \quad 0 < x < l.$$

Home: Ex. 20, Ex. 24(c, e),

$$u_t = a^2 u_{xx} + 1 + 2t, \quad 0 < x < 1, t > 0,$$

$$u(0, t) = u(1, t) = 0, \quad t > 0,$$

$$u(x, 0) = 0, \quad 0 < x < 1.$$

$$\left[u(x, t) = \frac{4}{a^4 \pi^5} \sum_{k=1}^{\infty} \frac{2 - a^2 \pi^2 (2k-1)^2}{(2k-1)^5} \left(e^{-a^2 \pi^2 (2k-1)^2 t} + \frac{2a^2 \pi^2 (2k-1)^2}{a^2 \pi^2 (2k-1)^2 - 2} t - 1 \right) \sin(2k-1)\pi x \right]$$

17.04.18. Initial boundary value problems on finite interval. Fourier method for the problems with nonhomogeneous boundary conditions:

(Exercises after Ch.7, p. 131)

Class: Ex. 27(c),

$$\begin{aligned}u_{tt} &= a^2 u_{xx}, & 0 < x < l, t > 0, \\u_x(0, t) &= e^t, & u_x(l, t) = e^t + 5, & t > 0, \\u(x, 0) &= 0, & u_t(x, 0) = 1 + x, & 0 < x < l.\end{aligned}$$

Home: Ex. 27(a),

$$\text{Ex. 29} \left[u(x, t) = 1 + \sin \pi x \left(-\frac{4}{\pi} - \frac{1}{2\pi^2} \right) \cos 2\pi t + \frac{1}{2\pi^2} + \sum_{n=2}^{+\infty} \frac{2((-1)^n - 1)}{\pi n} \sin \pi n x \cos 2\pi n t \right],$$

$$\text{Ex.35} \left[u(x, t) = \frac{\sin x}{4} (e^{-t} - 2te^{-t} + 3) + \frac{\sin 3x}{36} (e^{-t} (\cos 2\sqrt{2}t + \sin 2\sqrt{2}t) - 1) \right].$$

24.04.18. Integral transform methods:

(Exercises after Ch.9, p. 162)

Class: Ex. 1(a, c, d), Ex. 2(c), Ex. 4(c), Ex. 10(d).

Home: Ex.1 (e), Ex. 4(b, f), Ex. 8(a, b), Ex. 10(a,c).