Exercises of Mathematical analysis I

1. Find the domain of the function $y = \log(-x) + \dfrac{1}{x+5}$.
2. Find the domain of the function $y = \sqrt{3-x} + \arcsin\dfrac{3-2x}{5}$.
3. Find the domain of the function $y = \sqrt{\sin x} + \sqrt{16-x^2}$.
4. Find the domain of the function $y = \ln\dfrac{x-5}{x^2-10x+24} - \sqrt[3]{x+5}$.
5. Find the range of the function $y = 1 - 2\sin x$.
6. Find the domain of the function $y = \sqrt{3+2x-x^2}$.
7. Find the inverse function of the function $y = x^2 - 4x + 3$.
8. Find the inverse function of the function $y = \dfrac{2^x}{1+2^x}$.
9. Find the inverse function of the function $y = 1 - \log(3+e^x)$.
10. Find the inverse function of the function $y = 4\arcsin\sqrt{1-x^2}$.
11. Transform the function $\log_2 y - \log_2(x-1) = 3$ to explicit form.
12. Transform the function $(1+x)\cos y - x^2 = 0$ to explicit form.
13. Draw the graph of the function $y = |x| - x$.
14. Is the function $y = x - \dfrac{x^3}{6} + \dfrac{x^5}{120}$ even, odd or neither?
15. Is the function $y = x(5^x - 5^{-x})$ even, odd or neither?
16. Is the function $y = x^4 - 2x^3 + x$ even, odd or neither?
17. Is the function $y = x \cdot \ln\dfrac{1-x}{1+x}$ even, odd or neither?
18. Find $f\left(\dfrac{1+x}{1-x}\right)$, if $f(x) = \dfrac{1+x}{1-x}$.
19. Find $f\{f[f(1)]\}$, if $f(x) = x^2 - 1$.

In exercises 20. - 39. evaluate the limits.

20. $\lim\limits_{n\to\infty} \dfrac{(n+1)^3 - (n-1)^3}{(n+1)^2 + (n-1)^2}$.
21. $\lim\limits_{n\to\infty} \dfrac{\left(\sqrt{n^2+1}+n\right)^2}{\sqrt[3]{n^6+1}}$.
22. $\lim\limits_{x\to\frac{1}{2}} \dfrac{8x^3-1}{6x^2-5x+1}$.
23. $\lim\limits_{x\to 4} \dfrac{x^3-2x^2-8x}{x^2-x-12}$.

24. $\lim\limits_{x\to 2}\left[\dfrac{1}{x(x-2)^2}-\dfrac{1}{x^2-3x+2}\right]$.

25. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$.

26. $\lim\limits_{x\to 7}\dfrac{2-\sqrt{x-3}}{x^2-49}$.

27. $\lim\limits_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$.

28. $\lim\limits_{x\to 0}\dfrac{x^2}{1-\sqrt{1-x^2}}$.

29. $\lim\limits_{x\to\infty}\dfrac{(x+1)(x+2)}{2x^2}$.

30. $\lim\limits_{x\to\infty}\dfrac{10+x^5}{1-2x^5}$.

31. $\lim\limits_{x\to\infty}\dfrac{1-2x^2+3x^4}{1+2x^3}$.

32. $\lim\limits_{x\to 0}\dfrac{\tan 2x}{\sin 5x}$.

33. $\lim\limits_{x\to 0}\dfrac{1-\cos^3 x}{x\sin 2x}$.

34. $\lim\limits_{x\to 0}\left(\dfrac{1}{\sin x}-\dfrac{1}{\tan x}\right)$.

35. $\lim\limits_{x\to\frac{\pi}{2}}\left(\dfrac{\pi}{2}-x\right)\tan x$.

36. $\lim\limits_{x\to\infty}\left(1+\dfrac{4}{x}\right)^{\frac{x}{2}}$.

37. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+2}{x^2-1}\right)^{x^2}$.

38. $\lim\limits_{x\to\infty}\left(\dfrac{2x-3}{2x+1}\right)^{\frac{x-1}{2}}$.

39. $\lim\limits_{x\to 0}(1+x)^{\frac{2}{x}}$.

40. Using the definition of the derivative, prove that $(\sqrt{x})'=\dfrac{1}{2\sqrt{x}}$.

41. Using the definition of the derivative, prove that $\left(\dfrac{1}{x}\right)'=-\dfrac{1}{x^2}$.

In exercises 42. - 50. find the derivative of the function and simplify, if possible.

42. $y = \dfrac{1-x^2}{x^2+x^3}$

43. $y = \log_3(x^2+2x+4)$

44. $y = x \cdot 10^{\sqrt{x}}$

45. $y = \ln\left(x+\sqrt{1+x^2}\right) - \sqrt{1+x^2}$

46. $y = \sqrt[11]{9+6\sqrt[5]{x^9}}$

47. $y = \ln(e^x \cos x + e^{-x}\sin x)$

48. $y = \dfrac{1}{2}(3-x)\sqrt{1-2x-x^2} + 2\arcsin\dfrac{x+1}{\sqrt{2}}$

49. $y = \dfrac{3x^2-1}{3x^3} + \ln\sqrt{1+x^2} + \arctan x$

50. $y = \dfrac{\sin^2 x}{1+\cot x} + \dfrac{\cos^2 x}{1+\tan x}$

51. Evaluate $z'(0)$, if $z(t) = \left(\sqrt{t^3}+1\right)t$.

52. The angle of rotation α of the belt drive depends on time as $\alpha = t^2+3t-5$. Evaluate the angular speed at $t=5$.

53. Find the slope of the tangent line of the graph of the function $y = \dfrac{8a^3}{4a^2+x^2}$ at the point with abscissa $x = 2a$.

54. Find y', if $x^4+y^4 = x^2y^2$.

55. Find y', if $y\sin x - \cos(x-y) = 0$.

56. Find y', if $2y\ln y = x$.

57. Find y', if $2^x + 2^y = 2^{x+y}$.

58. Find y', if $y = x^{\frac{1}{x}}$.

59. Find y', if $y = \left(\dfrac{x}{1+x}\right)^x$.

60. Find y', if $y = \dfrac{\sqrt{x-2}}{(x+3)^3\sqrt[5]{x^2}}$.

61. Find $\dfrac{dy}{dx}$, if $x = t(1-\sin t)$, $y = t\cos t$.

62. Find the slope of the tangent line of the ellipse $x = 2\cos t$, $y = \sin t$ at the point $A\left(1; -\dfrac{\sqrt{3}}{2}\right)$.

63. Express the differential dy of the function $y = xe^{2x}$.

64. Evaluate the differential and the increment of the function $y = \ln \dfrac{x}{x^2+1}$, if $x = 2$ and $\Delta x = \dfrac{1}{30}$.

65. Using the differential of function, evaluate the approximate value of $\ln 1,01$.

66. Using the differential of function, evaluate the approximate value of $\sqrt[4]{16,64}$.

67. Find y'', if $y = \sqrt{1+x^2}$.

68. Find y'', if $y = x(\sin \ln x + \cos \ln x)$.

69. Find $y^{(n)}$, if $y = \dfrac{x}{x+1}$.

70. Evaluate $f^{IV}(1)$, if $f(x) = x^6 - 4x^3 + 4$.

71. Find y'', if $e^{x+y} = xy$.

72. Find $\dfrac{d^2y}{dx^2}$, if $x = \ln t$, $y = t^2 - 1$.

In exercises 73. - 80. evaluate the limits using the L'Hospital's rule.

73. $\lim\limits_{x\to 2} \dfrac{\sqrt[3]{x} - \sqrt[3]{2}}{\sqrt{x} - \sqrt{2}}$

74. $\lim\limits_{x\to 0} \dfrac{e^{ax} - e^{bx}}{2x}$

75. $\lim\limits_{x\to \infty} \dfrac{\pi - 2\arctan x}{\ln\left(1 + \dfrac{1}{x}\right)}$

76. $\lim\limits_{x\to \infty} x^3 e^{-x}$

77. $\lim\limits_{x\to 1} \left(\dfrac{1}{\ln x} - \dfrac{x}{\ln x}\right)$

78. $\lim\limits_{x\to 0} x^{\sin x}$

79. $\lim\limits_{x\to 0} \left(\dfrac{1}{x}\right)^x$

80. $\lim\limits_{x\to 0} (e^x + x)^{\frac{1}{x}}$.

81. Using the Taylor's formula, expand the function $f(x) = x^5 - 3x^3 + 1$ in powers $x - 1$.

82. Compose the second order Taylor's formula of the function $y = \sin^2 x$ in the neighborhood of $x_0 = 0$. Using the polynomial obtained, evaluate the approximate value of $\sin^2 0,3$ and with help of the remainder estimate the greatest possible error.

83. Compose the third order Taylor's formula of the function $y = x^3 \ln x$ in the neighborhood of $x_0 = 1$.

84. Find the intervals of increase and decrease of the function $y = \dfrac{x}{\ln x}$.

85. In given closed interval $[0; 2\pi]$ find the intervals in which the function $y = 2\sin x + \cos 2x$ is increasing and decreasing.

86. Find the local extremums of the function $y = x - \ln(1 + x)$.

87. Find the local extremums of the function $y = (x - 5)^2 \sqrt[3]{(x + 1)^2}$.

88. Find the local extremums of the function $y = x \sin x + \cos x - \dfrac{1}{4}x^2$ in the closed interval $\left[-\dfrac{\pi}{2}; \dfrac{\pi}{2}\right]$.

89. Find the greatest and the least value of the function $y = \dfrac{x - 1}{x + 1}$ in the closed interval $[0; 4]$.

90. Find the greatest and the least value of the function $y = \sin 2x - x$ in the closed interval $\left[-\dfrac{\pi}{2}; \dfrac{\pi}{2}\right]$.

91. Find the domains of convexity and concavity and the inflection points of the graph of the function $y = \dfrac{x^3}{x^2 + 3}$.

92. Find the domains of convexity and concavity and the inflection points of the graph of the function $y = e^{-x^2}$.

93. Find the asymptotes of the graph of the function $y = \dfrac{3x^2 - 2x}{x - 1}$.

94. Find the asymptotes of the graph of the function $y = \dfrac{x}{2} + \dfrac{2}{x}$.

95. For the given function $y = \dfrac{x^3}{3} + \dfrac{1}{x}$ find the domain, zeros, the local extrema, the intervals where function is increasing and decreasing, the domains of convexity and concavity and the inflection points, the asymptotes of the graph of the function. Is this function even or odd? Draw the graph of this function.

In exercises 96. - 127. find the indefinite integral

96. $\displaystyle\int \frac{\sqrt[3]{x^2} - \sqrt[4]{x}}{\sqrt{x}} dx$

97. $\displaystyle\int \frac{(1 + 2x^2)dx}{x^2(1 + x^2)}$

98. $\displaystyle\int \frac{1 + \cos^2 x}{1 + \cos 2x} dx$

99. $\int \frac{dx}{x^2-5}$

100. $\int \sqrt{5-2x}dx$

101. $\int \frac{x^3dx}{\sqrt{x^4+3}}$

102. $\int \tan x dx$

103. $\int \sin^4 x \cos x dx$

104. $\int \frac{e^x dx}{e^x+2}$

105. $\int \frac{dx}{x \ln x}$

106. $\int \frac{xdx}{x^4+1}$

107. $\int \frac{dx}{x\sqrt{1-\ln^2 x}}$

108. $\int \frac{1+x}{\sqrt{1-x^2}}dx$

109. $\int (x+2)\sin 2x dx$

110. $\int x3^x dx$

111. $\int \ln(x^2+1)dx$

112. $\int \arccos x dx$

113. $\int \frac{2x+3}{3x+2}dx$

114. $\int \frac{x^3dx}{x+1}$

115. $\int \frac{dx}{6x^3-7x^2-3x}$

116. $\int \frac{x^5+x^4-8}{x^3-4x}dx$

117. $\int \frac{x^2+1}{(x^2-1)(x^2-4)}dx$

118. $\int \frac{3x-2}{x(x^2+1)}dx$

119. $\int \frac{dx}{x^3+1}$

120. $\int \frac{dx}{4+5\cos x}$

121. $\int \frac{\tan x}{1-2\tan x}dx$

122. $\int \frac{dx}{\cos^6 x}$

123. $\int \frac{\sin^3 x dx}{\cos^4 x}$

124. $\int \sin^2 x dx$

125. $\int \frac{dx}{\sqrt{x}(x+1)}$

126. $\int \frac{2+x}{\sqrt[3]{3-x}}dx$

127. $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

In exercises 128. - 138. evaluate the definite integral

128. $\int_0^{16} \frac{dx}{\sqrt{x+9}-\sqrt{x}}$

129. $\int_1^2 \frac{e^{\frac{1}{x}}dx}{x^2}$

130. $\int_1^{e^3} \frac{dx}{x\sqrt{1+\ln x}}$

131. $\int_0^1 \frac{dx}{x^2+4x+5}$

132. $\int_1^2 \frac{dx}{x+x^2}$

133. $\int_0^{\frac{\pi}{2}} \cos^5 x \sin 2x dx$

134. $\int_0^{\pi} x^3 \sin x dx$

135. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{x dx}{\sin^2 x}$

136. $\int_0^{e-1} \ln(x+1)dx$

137. $\int_0^1 \frac{\sqrt{x}dx}{1+x}$

138. $\int_3^{29} \frac{\sqrt[3]{(x-2)^2}dx}{3+\sqrt[3]{(x-2)^2}}$

In exercises 139. - 145. evaluate the improper integral

139. $\int_0^\infty e^{-2x}dx$

140. $\int_{-\infty}^\infty \frac{dx}{x^2+2x+2}$

141. $\int_0^\infty \frac{x}{(1+x)^3}dx$

142. $\int_0^\infty x\sin x dx$

143. $\int_1^2 \frac{xdx}{\sqrt{x-1}}$

144. $\int_0^1 x\ln x dx$

145. $\int_1^2 \frac{dx}{\sqrt{(x-1)(2-x)}}$

146. Determine the area between the parabolas $y^2+8x=16$ and $y^2-24x=48$.

147. Determine the area of the region bounded by the astroid $x=a\cos^3 t$, $y=a\sin^3 t$.

148. Determine the area of the region bounded by the limaçon of Pascal $\varrho = 2a(2+\cos\varphi)$.

149. Determine the length of the arc of the curve $y=\ln(1-x^2)$ between $x=0$ and $x=\frac{1}{2}$.

150. Determine the length of the curve $y=\sqrt{x-x^2}+\arcsin\sqrt{x}$.

151. Determine the length of the arc of the curve $x=a(\cos t+t\sin t)$, $y=a(\sin t-t\cos t)$ between $t=0$ and $t=\pi$.

152. Determine the length of the arc of the hyperbolic spiral $\varrho\varphi=1$ between $\varphi=\frac{3}{4}$ and $\varphi=\frac{4}{3}$.

153. Determine the equations of the tangent and the normal of the function $y=\ln x$ at the point with abscissa $x_0=1$. Draw the graph of the function and the tangent and normal obtained.

154. Determine the equations of the tangent and the normal of the function $y = \dfrac{8a^3}{x^2+4a^2}$ at the point with abscissa $x_0 = 2a$.

155. Determine the equations of the tangent and the normal of the hyperbola $x^2 - y^2 = 5$ at the point $(3; 2)$.

156. Determine the equations of the tangent and the normal of the curve $x = \sin t$, $y = \cos 2t$ at the point in which the value of the parameter is $t = \dfrac{\pi}{6}$.

157. Determine the equations of the tangent and the normal of the ellipse $x = 3\cos t$, $y = 4\sin t$ at the point $\left(\dfrac{3\sqrt{2}}{2}; 2\sqrt{2}\right)$.

Answers

1. $X = (-\infty; -5) \cup (-5; 0)$; 2. $X = [-1; 3]$; 3. $X = [-4; -\pi] \cup [0; \pi]$; 4: $X = (4; 5) \cup (6; \infty)$; 5. $Y = [-1; 3]$ 6. $Y = [0; 2]$; 7. $y = 2 \pm \sqrt{1+x}$; 8. $y = \log_2 \dfrac{x}{1-x}$; 9. $y = \ln(10^{1-x} - 3)$; 10. $y = \pm\cos\dfrac{x}{4}$ $(0 \le x \le 2\pi$; 11. $y = 8x - 8$; 12. $y = \arccos\frac{x^2}{1+x}$; 14. odd; 15. even; 16. neither; 17. even; 18. $-\dfrac{1}{x}$; 19. 0; 20. 3; 21. 4; 22. 6; 23. $3\dfrac{3}{7}$; 24. ∞; 25. 1; 26. $-\dfrac{1}{56}$; 27. $\dfrac{1}{2\sqrt{x}}$; 28. 2; 29. $\dfrac{1}{2}$; 30. $-\dfrac{1}{2}$; 31. ∞; 32. $\dfrac{2}{5}$; 33. $\dfrac{3}{4}$; 34. 0; 35. 1; 36. e^2; 37. e^3; 38. $\dfrac{1}{e}$; 39. e^2; 42. $\dfrac{x-2}{x^3}$; 43. $\dfrac{2x+2}{(x^2+2x+4)\ln 3}$; 44. $10^{\sqrt{x}}\left(1 + \dfrac{\sqrt{x}\ln 10}{2}\right)$; 45. $\dfrac{1-x}{\sqrt{1+x^2}}$; 46. $\dfrac{54\sqrt[5]{x^4}}{55 \cdot \sqrt[11]{\left(9+6\sqrt[5]{x^9}\right)^{10}}}$; 47. $\dfrac{(e^x + e^{-x})(\cos x - \sin x)}{e^x\cos x + e^{-x}\sin x}$; 48. $\dfrac{x^2}{\sqrt{1-2x-x^2}}$; 49. $\dfrac{x^5+1}{x^4(1+x^2)}$; 50. $-\cos 2x$; 51. 1; 52. $13\frac{rad}{s}$; 53. $-\dfrac{1}{2}$; 54 $\dfrac{x}{y} \cdot \dfrac{y^2 - 2x^2}{2y^2 - x^2}$; 55. $\dfrac{y\cos x + \sin(x-y)}{\sin(x-y) - \sin x}$; 56. $\dfrac{1}{2(1+\ln y)}$; 57. $2^{x-y} \cdot \dfrac{2^y - 1}{1 - 2^x}$; 58. $x^{\frac{1}{x}-2}(1 - \ln x)$;

59. $\left(\dfrac{x}{1+x}\right)^x \left(\dfrac{1}{1+x} + \ln\dfrac{x}{1+x}\right)$; 60. $\dfrac{\sqrt{x-2}}{(x+3)^3 \cdot \sqrt[5]{x^2}}\left[\dfrac{1}{2(x-2)} - \dfrac{3}{x+3} - \dfrac{2}{5x}\right]$;

61. $\dfrac{\cos t - t\sin t}{1 - \sin t - t\cos t}$; 62. $\dfrac{\sqrt{3}}{6}$; 63. $e^{2x}(1+2x)dx$; 64. $dy = -0,01$, $\Delta y = -0,0100044$; 65. $0,01$; 66. $2,02$; 67. $\dfrac{1}{(1+x^2)\sqrt{1+x^2}}$;

68. $-\dfrac{2\sin\ln x}{x}$; 69. $(-1)^{n+1} \cdot \dfrac{n!}{(x+1)^{n+1}}$; 70. 360; 71. $-\dfrac{y\left[(x-1)^2 + (y-1)^2\right]}{x^2(y-1)^3}$;

72. $4t^2$; 73. $\dfrac{2}{3\cdot\sqrt[6]{2}}$; 74. $\dfrac{a-b}{2}$; 75. 2; 76. 0; 77. -1; 78. 1; 79. 1; 80. e^2; 81. $-1-4(x-1)+(x-1)^2+7(x-1)^3+5(x-1)^4+(x-1)^5$; 82. $x^2+R_2(x)$, where $R_2(x)=-\dfrac{2x^3\sin 2\Theta x}{3}$; 83. $x-1+\dfrac{5}{2}(x-1)^2+\dfrac{11}{6}(x-1)^3+R_3(x)$, where $R_3(x)=\dfrac{(x-1)^4}{4[1+\Theta(x-1)]}$; 84. $X\uparrow=(e;\infty)$, $X\downarrow=(0;1)$, $X\downarrow=(1;e)$; 85. $X\uparrow=\left(0;\dfrac{\pi}{6}\right)$, $X\uparrow=\left(\dfrac{\pi}{2};\dfrac{5\pi}{6}\right)$, $X\uparrow=\left(\dfrac{3\pi}{2};2\pi\right)$, $X\downarrow=\left(\dfrac{\pi}{6};\dfrac{\pi}{2}\right)$, $X\downarrow=\left(\dfrac{5\pi}{6};\dfrac{3\pi}{2}\right)$; 86. At $x=0$ local minimum 87. At $x=-1$ and $x=5$ local minima, at $x=0,5$ local maximum 88. At $x=0$ local minimum. At $x=\pm\dfrac{\pi}{3}$ local maximum 89. $y_{min}=y(0)=-1$, $y_{max}=y(4)=\dfrac{3}{5}$; 90. $y_{max}=y\left(-\dfrac{\pi}{2}\right)=\dfrac{\pi}{2}$, $y_{min}=y\left(\dfrac{\pi}{2}\right)=-\dfrac{\pi}{2}$; 91. $\hat{X}=\left(-\infty;-\dfrac{3}{\sqrt{5}}\right)$, $\hat{X}=\left(0;\dfrac{3}{\sqrt{5}}\right)$, $\breve{X}=\left(-\dfrac{3}{\sqrt{5}};0\right)$, $\breve{X}=\left(\dfrac{3}{\sqrt{5}};\infty\right)$; 92. $\hat{X}=\left(-\dfrac{1}{\sqrt{2}};\dfrac{1}{\sqrt{2}}\right)$, $\breve{X}=\left(-\infty;-\dfrac{1}{\sqrt{2}}\right)$, $\breve{X}=\left(\dfrac{1}{\sqrt{2}};\infty\right)$; 93. $x=1$, $y=3x+1$; 94. $x=0$, $y=\dfrac{x}{2}$; 95. $X=(-\infty;0)\cup(0;\infty)$, no zeros, at $x=-1$ local maximum, at $x=1$ local minimum $X\uparrow=(-\infty;-1)$, $X\uparrow=(1;\infty)$, $X\downarrow=(-1;0)$, $X\downarrow=(0;1)$, $\hat{X}=(-\infty;0)$, $\breve{X}=(0;\infty)$, no inflection points, vertical asymptote $x=0$, no inclined asymptotes; 96. $\dfrac{6x\sqrt[6]{x}}{7}-\dfrac{4\sqrt[4]{x^3}}{3}+C$; 97. $\arctan x-\dfrac{1}{x}+C$; 98. $\dfrac{1}{2}(\tan x+x)+C$; 99. $\dfrac{1}{2\sqrt{5}}\ln\left|\dfrac{\sqrt{5}-x}{\sqrt{5}+x}\right|+C$; 100. $\dfrac{(2x-5)\sqrt{5-2x}}{3}+C$; 101. $\dfrac{1}{2}\sqrt{x^4+3}+C$; 102. $-\ln|\cos x|+C$; 103. $\dfrac{\sin^5 x}{5}+C$; 104. $\ln(e^x+2)+C$; 105. $\ln|\ln x|+C$; 106. $\dfrac{1}{2}\arctan x^2+C$; 107. $\arcsin\ln x+C$; 108. $\arcsin x-\sqrt{1-x^2}+C$; 109. $-\dfrac{(x+2)\cos 2x}{2}+\dfrac{\sin 2x}{4}+C$; 110. $\dfrac{x3^x}{\ln 3}-\dfrac{3^x}{\ln^2 3}+C$; 111. $x\ln(x^2+1)-2x+2\arctan x+C$; 112. $x\arccos x-\sqrt{1-x^2}+C$; 113. $\dfrac{2x}{3}+\dfrac{5}{9}\ln|3x+2|+C$; 114. $\dfrac{x^3}{3}-\dfrac{x^2}{2}+x-\ln|x+1|+C$; 115. $\dfrac{2}{33}\ln|2x-3|+\dfrac{3}{11}\ln|3x+1|-\dfrac{1}{3}\ln|x|+C$; 116. $\dfrac{x^3}{3}+\dfrac{x^2}{2}+4x+2\ln|x|+5\ln|x-2|-3\ln|x+2|+C$; 117. $\dfrac{1}{3}\ln\left|\dfrac{x+1}{x-1}\right|+\dfrac{5}{12}\ln\left|\dfrac{x-2}{x+2}\right|+C$; 118. $\ln\dfrac{x^2+1}{x^2}+3\arctan x+C$; 119. $\dfrac{1}{3}\ln|x+1|-\dfrac{1}{6}\ln(x^2-x+1)+\dfrac{1}{\sqrt{3}}\arctan\dfrac{2x-1}{\sqrt{3}}+C$; 120. $\dfrac{1}{3}\ln\left|\dfrac{\tan\frac{x}{2}+3}{\tan\frac{x}{2}-3}\right|+C$; 121. $-\dfrac{1}{5}\ln|2\sin x-\cos x|-\dfrac{2x}{5}+C$; 122. $\tan x+\dfrac{2}{3}\tan^3 x+\dfrac{1}{5}\tan^5 x+C$; 123. $\dfrac{1}{3\cos^3 x}-\dfrac{1}{\cos x}+C$; 124. $\dfrac{1}{2}x-\dfrac{1}{4}\sin 2x+C$; 125. $2\arctan\sqrt{x}+C$;

126. $\frac{3}{5}(3-x)\sqrt[3]{(3-x)^2} - \frac{15}{2}\sqrt[3]{(3-x)^2} + C$; 127. $2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\ln(\sqrt[6]{x}+1) + C$; 128. 12; 129. $e - \sqrt{e}$; 130. 2; 131. $\arctan 3 - \arctan 2$; 132. $\ln\frac{4}{3}$; 133. $\frac{2}{7}$; 134. $\pi^3 - 6\pi$; 135. $\frac{\pi(9-4\sqrt{3})}{36} + \frac{1}{2}\ln\frac{3}{2}$; 136. 1; 137. $2 - \frac{\pi}{2}$; 138. $8 + \frac{3\sqrt{3}}{2}\pi$; 139. $\frac{1}{2}$; 140. π; 141. $\frac{1}{2}$; 142. diverges; 143. $\frac{8}{3}$; 144. $-\frac{1}{4}$; 145. π; 146. $\frac{32}{3}\sqrt{6}$; 147. $\frac{3}{8}\pi a^2$; 148. $18\pi a^2$; 149. $\ln 3 - \frac{1}{2}$; 150. 2; 151. $\frac{\pi^2 a}{2}$; 152. $\ln\frac{3}{2} + \frac{5}{12}$; 153. $y = x-1$ and $y = 1-x$; 154. $y = -\frac{1}{2}x + 2a$ and $y = 2x - 3a$; 155. $y = \frac{3}{2}x - \frac{5}{2}$ and $y = -\frac{2}{3}x + 4$; 156. $y = -2x + \frac{3}{2}$ and $y = \frac{1}{2}x + \frac{1}{4}$; 157. $y = -\frac{4}{3}x + 4\sqrt{2}$ and $y = \frac{3}{4}x + \frac{7\sqrt{2}}{8}$