# Exercises of Mathematical analysis II

In exercises 1. - 8. represent the domain of the function by the inequalities and make a sketch showing the domain in $xy$-plane.

1. $z = \sqrt{x - \sqrt{y}}$.

2. $z = \arcsin \dfrac{y+2}{x-1} + \ln y$.

3. $z = \sqrt{\sin \pi(x^2 + y^2)}$.

4. $z = \ln x - \ln \sin y$.

5. $z = \sqrt{4 - x^2} + \ln(y^2 - 4)$

6. $z = \sqrt{\arcsin \dfrac{x}{y}}$

7. $z = (y + \sqrt{y})\sqrt{\cos x}$

8. $z = \sqrt{x^2 + y^2 - 1} - 2\ln(9 - x^2 - y^2)$

9. Are the functions

$$z = \sqrt{x \sin y} \quad \text{and} \quad z = \sqrt{x}\sqrt{\sin y}$$

identical? Why?

10. Are the functions

$$z = \ln xy \quad \text{and} \quad z = \ln x + \ln y$$

identical? Why?

11. $f(x, y) = \dfrac{3x^2 - y}{2x + y}$. Evaluate $\lim\limits_{x\to 0}\lim\limits_{y\to 0} f(x, y)$ and $\lim\limits_{y\to 0}\lim\limits_{x\to 0} f(x, y)$. Does $\lim\limits_{(x,y)\to(0,0)} f(x, y)$ exist?

Evaluate the limit

12. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$

13. $\lim_{(x,y)\to(0,0)} \dfrac{\sqrt{x^2y^2+1}-1}{x^2+y^2}$

14. $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$

15. $\lim_{(x,y)\to(0,0)} \dfrac{xy(x^2-y^2)}{x^2+y^2}$

16. $\lim_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$

17. $\lim_{(x,y)\to(0,0)} \dfrac{\sin(x^3+y^3)}{x^2+y^2}$

In exercises 18. - 28. find all first-order partial derivatives of the given function.

18. $z = x^2\sqrt[3]{y} + \dfrac{\sqrt{x}}{\sqrt[4]{y}}$

19. $z = \ln\tan\dfrac{x}{y}$

20. $z = e^{-\frac{x}{y}}$.

21. $z = \sin xy - \cos\dfrac{y}{x}$.

22. $z = \ln(x + \sqrt{x^2+y^2})$

23. $z = \arctan\dfrac{y}{\sqrt{x}}$.

24. $z = xy\ln(x+y)$.

25. $w = \ln(xy + \ln z)$

26. $w = \tan(x^2+y^3+z^4)$.

27. $w = x^{y^z}$.

28. $z = (x^2+y^2)\dfrac{1-\sqrt{x^2+y^2}}{1+\sqrt{x^2+y^2}}$.

29. Evaluate the partial derivatives of $z = \arcsin\dfrac{x}{\sqrt{x^2+y^2}}$ at $(1;-2)$.

30. $z = \dfrac{x\cos y - y\cos x}{1+\sin x+\sin y}$. Evaluate $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, if $x = y = 0$.

31. $w = \ln(1 + x + y^2 + z^3)$. Evaluate $\dfrac{\partial w}{\partial x} + \dfrac{\partial w}{\partial y} + \dfrac{\partial w}{\partial z}$ at the point $x = y = z = 1$

32. $z = \ln(x^2 - y^2)$; prove that $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} - \dfrac{2}{x+y} = 0$

33. For the function $z = xy + x\arctan\dfrac{y}{x}$ prove that et $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = xy + z$.

34. Find the total differential of the function $z = \arcsin\dfrac{x}{y}$.

35. Find the total differential of the function $z = \sin\dfrac{x}{y}\cos\dfrac{y}{x}$.

36. Find the total differential of the function $z = \ln\sin\dfrac{x}{y}$.

37. Find the total differential of the function $w = x^{yz}$.

38. Find the total differential of the function $z = \dfrac{xy}{x^2 - y^2}$, if $x = 2$, $y = 1$, $\Delta x = 0{,}01$ and $\Delta y = 0{,}03$.

39. Evaluate the increment $\Delta z$ and total differential $dz$ of the function $z = \dfrac{x+y}{x-y}$, if $x = -3$, $y = 7$, $\Delta x = -\dfrac{1}{3}$ and $\Delta y = \dfrac{1}{4}$.

40. Evaluate the increment $\Delta z$ and total differential $dz$ of the function $z = xy + \dfrac{x}{y}$, if $x$ changes from $-1$ to $-0{,}8$ and $y$ from 2 to $2{,}2$.

41. Using total differential, compute the approximate value of $1{,}96^3 \cdot 2{,}03^5$.

42. Using total differential, compute the approximate value of $\dfrac{\sqrt{82}}{\sqrt[3]{28}}$.

43. Using total differential, compute the approximate value of $\arcsin\dfrac{\sqrt{1{,}04}}{2{,}04}$.

44. Using total differential, compute the approximate value of $\ln\left(\sqrt[5]{0{,}98} + \sqrt[4]{1{,}04} - 1\right)$.

45. Find $\dfrac{dy}{dx}$, if $x^2y^2 - x^4 - y^4 = a^2$.

46. Find $\dfrac{dy}{dx}$, if $2y^3 + 3x^2y + \ln x = 0$ and evaluate it at $x = 1$.

47. Find $\dfrac{dy}{dx}$, if $y = \sqrt{x}\ln\dfrac{x}{y}$ and evaluate it at the point $(e^2; e)$.

48. Find $\dfrac{dy}{dx}$, if $x^y = y^x$ and evaluate it at the point $(1; 1)$.

49. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, if $x^2 - 2y^2 + z^2 - 4x + 2z - 5 = 0$.

50. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, if $z = \cos xy - \sin xz$ and evaluate these at the point $\left(\dfrac{\pi}{2}; 1; 0\right)$.

51. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, if $xyz = e^z$ and evaluate these at the point $(e^{-1}; -1; -1)$.

52. Find the total differential of $z$, if $z$ is determined by the equation $\cos^2 x + \cos^2 y + \cos^2 z = 1$.

53. Find $\dfrac{dz}{dx}$, if $z = \arctan(xy + 1)$ and $y = \ln x$.

54. Find $\dfrac{dz}{dx}$, if $z = \arcsin\dfrac{x}{y}$ and $y = \sqrt{x^2 + 4}$.

55. Find $\dfrac{dz}{dt}$, if $z = \tan(3t + 2x^2 - y)$, $x = \dfrac{1}{t}$ and $y = \sqrt{t}$.

56. Find $\dfrac{du}{dx}$, if $u = \dfrac{e^{2x}}{5}(y - z)$, $y = 2\sin x$ and $z = \cos x$.

57. Find $\dfrac{dw}{dx}$, if $w = \sqrt{x^2 + u^2 + v^2}$, $u = \sin x$ and $v = e^x$.

58. Find $\dfrac{dz}{dt}$, if $z = \arcsin\dfrac{y}{x}$, $x = \sin t$ and $y = t^2$.

59. Find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$, if $z = \sqrt{x^2 + y^2}$, $x = u\cos v$ and $y = v\cos u$.

60. Find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$, if $z = \ln(x^2 + y^2)$, $x = u\cosh v$ and $y = v\sinh u$.

61. Find the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ of the function $z = \arctan uv$, if $u = xy$ and $v = x - y$.

62. Prove that $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x}$, if $z = e^x(\cos y + x \sin y)$.

63. Find $\dfrac{\partial^2 z}{\partial x^2}$, $\dfrac{\partial^2 z}{\partial y^2}$ and $\dfrac{\partial^2 z}{\partial x \partial y}$, if $z = \arcsin(xy)$.

64. Find $\dfrac{\partial^3 w}{\partial x \partial y \partial z}$, if $w = e^{xyz}$.

65. Evaluate all second order derivatives of the function $z = \dfrac{x}{y^2}$ at the point $(-1; -2)$.

66. Evaluate all second order derivatives of the function $z = \arcsin \dfrac{x}{\sqrt{x^2 + y^2}}$ at the point $(1; -2)$.

67. Evaluate

$$\frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2$$

at the point $(0; -2)$, if $z = \dfrac{\cos x^2}{y}$.

68. For the function $z = \ln(e^x + e^y)$ prove that $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 1$ and $\dfrac{\partial^2 z}{\partial x^2} \dfrac{\partial^2 z}{\partial y^2} - \left(\dfrac{\partial^2 z}{\partial x \partial y}\right)^2 = 0$.

69. Prove that the function $z = \dfrac{x^2 y^2}{x + y}$ satisfies the equation

$$x \frac{\partial^2 z}{\partial x^2} + y \frac{\partial^2 z}{\partial x \partial y} = 2 \frac{\partial z}{\partial x}$$

70. Find the canonical equations of the tangent line of spatial curve $x = 1 + \sin t$, $y = 2t - \cos t$, $z = 3 + t^2$ at $t = 0$.

71. Find the canonical equations of the tangent line of screw line $x = 2 \cos t$, $y = 2 \sin t$, $z = 4t$ at $t = \dfrac{\pi}{4}$

72. Find the canonical equations of the tangent line of spatial curve $x = t - \sin t$, $y = 1 - \cos t$, $z = 4 \sin \dfrac{t}{2}$ at $\left(\dfrac{\pi}{2} - 1; 1; 2\sqrt{2}\right)$.

73. On the curve $y = x^2$, $z = x^3$ find the points at which the tangent line is parallel to the plane $x + 2y + z = -1$.

74. Find the equation of the tangent plane and the canonical equations of the normal line for the surface $z = \arctan\frac{y}{x}$ at the point $\left(2; -2; -\frac{\pi}{4}\right)$.

75. Find the equation of the tangent plane and the canonical equations of the normal line for the surface $z = \sqrt{x^2 + y^2}$ at $(3; -4; 5)$.

76. Find the equation of the tangent plane and the canonical equations of the normal line for the surface $z = \cos\frac{y}{x}$ at $(-1; -\pi; -1)$.

77. Find the equation of the tangent plane and the canonical equations of the normal line for the surface $x^2y^2 + 2x + z^3 = 16$ at the point $x = 2$ and $y = 1$.

78. Prove that the surfaces $x + 2y - \ln z = -4$ and $x^2 - xy - 8x + z = -5$ have the same tangent plane at the point $(2; -3; 1)$.

79. Find the gradient vector for the scalar field $z = x - 3y + \sqrt{3xy}$ at the point $(3; 4)$.

80. Find the points at which the gradient vector of the scalar field $z = \ln\left(x + \frac{1}{y}\right)$ is $\overrightarrow{a} = \left(1; -\frac{16}{9}\right)$.

81. Find the gradient vector for the scalar field $w = \arcsin\frac{\sqrt{x^2 + y^2}}{z}$ at the point $(1; 1; 2)$.

82. Find the directional derivative of the function $z = \arctan\frac{y}{x} - \frac{4y}{x}$ at the point $(1; \sqrt{3})$ in direction the point $(2; 3\sqrt{3})$.

83. Find the directional derivative of the function $w = xyz$ at the point $A(-2; 1; 3)$ in the direction of $\overrightarrow{s} = (4; 3; 12)$.

84. Find the directional derivative of the function $w = x^2y^2 - z^2 + 2xyz$ at the point $B(1; 1; 0)$ in direction forming with coordinate axes the angles $60°$, $45°$ and $60°$ respectively.

85. Find the greatest increase of the function $z = \ln(x^2 + y^2)$ at the point $C(-3; 4)$

86. Find the greatest value of the derivative of function given by the equation $x^2 + y^3 - z^2 - 1 = 0$ at the point $(3; 2; 4)$.

87. Find the steepest ascent of the surface $z = \arctan \dfrac{y}{x}$ at the point $(1; 1)$.

88. Find the direction of greatest increase of the function $f(x, y, z) = x \sin z - y \cos z$ at the origin.

89. Find the divergence and curl of the vector field $\overrightarrow{F} = \left( \dfrac{x}{y}; \dfrac{y}{z}; \dfrac{z}{x} \right)$.

90. Find the divergence and curl of the vector field $\overrightarrow{F} = (\ln(x^2 - y^2); \arctan(z - y); xyz)$.

91. Find the divergence and curl of the vector field $\overrightarrow{F} = \operatorname{grad} w$, if $w = \ln(x + y - z)$.

92. Find the divergence and curl of the vector field $\overrightarrow{F} = \operatorname{rot} \overrightarrow{G}$, if $\overrightarrow{G} = (x^2 y; y^2 z; x^2 z)$.

93. Find the local extrema of the function $z = 4x^2 - xy + 9y^2 + x - y$ and determine their type.

94. Find the local extremum points of the function $z = x^3 y^2 (12 - x - y)$, satisfying the conditions $x > 0$ and $y > 0$ and determine their type.

95. Find the local extrema of the function $z = x^2 + xy + y^2 + \dfrac{1}{x} + \dfrac{1}{y}$ and determine their type.

96. Find the local extrema of the function $z = e^x (x^2 + y^2)$ and determine their type.

97. Find the local extrema of the function $z = x^3 + y^3 - 3xy$ and determine their type.

98. Find the greatest and the least value of the function $z = x^2 + 2xy - 4x + 8y$ in the rectangle bounded by $x = 0$, $y = 0$, $x = 1$ and $y = 2$.

99. Find the greatest and the least value of the function $z = x^2 - y^2$ in the circle $x^2 + y^2 \leq 4$.

100. Find the greatest and the least value of the function $z = \sin x + \sin y + \sin(x + y)$ in the quadrate $0 \leq x \leq \dfrac{\pi}{2}$, $0 \leq y \leq \dfrac{\pi}{2}$.

101. Find the extremal values of the function $z = \frac{1}{x} + \frac{1}{y}$ under the condition $x + y = 2$.

102. Find the extremal values of the function $z = a\cos^2 x + b\cos^2 y$ under the condition $y - x = \frac{\pi}{4}$.

103. Find the extremal values of the function $w = x + y + z$ under the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

104. The sum of three edges of the rectangular box, passing one vertex is 1 m. Find the dimensions of this rectangular box so that the volume is the greatest.

105. Find the point on the parabola $y = 3x^2 - 2$ closest to the point$P_0(0; 2)$.

106. Find the point in the plane $3x - 2z = 0$ so that the sum of squares of distances from the points $A(1; 1; 1)$ and $B(2; 3; 4)$ is the least.

107. Evaluate the double integral $\iint\limits_D (x^2 + y^2)dxdy$, if $D$ is the quadrate $0 \le x \le 1$ and $1 \le y \le 2$.

108. Evaluate the double integral $\iint\limits_D \frac{dxdy}{(x+y)^2}$, if $D$ is the quadrate $1 \le x \le 2$ and $3 \le y \le 4$.

109. Evaluate the double integral $\int\limits_1^2 dx \int\limits_x^{x\sqrt{3}} xydy$.

110. Evaluate the double integral $\int\limits_0^1 dx \int\limits_{-x}^{x+1} (xy + y)dy$.

111. Evaluate the double integral $\int\limits_0^1 dx \int\limits_{x^2}^{\sqrt{x}} (x^2 + y^2)dy$.

112. Sketch the domain of integration and determine the limits of integration for $\iint\limits_D f(x; y)dxdy$, if $D$ is the region bounded by the line $y = 0$ and the parabola $y = 1 - x^2$.

113. Sketch the domain of integration and determine the limits of integration for $\iint\limits_D f(x;y)dxdy$, if $D$ is the parallelogram bounded by the lines $y=0$, $y=a$, $y=x$ and $y=x-2a$.

114. Sketch the domain of integration and determine the limits of integration for $\iint\limits_D f(x;y)dxdy$, if $D$ is the region bounded by $y=\dfrac{2}{1+x^2}$ and $y=x^2$.

115. Sketch the domain of integration and change the order of integration for $\int\limits_0^1 dx \int\limits_{x^3}^{\sqrt{x}} f(x;y)dy$.

116. Sketch the domain of integration and change the order of integration for $\int\limits_{-1}^1 dx \int\limits_{-\sqrt{1-x^2}}^{x+1} f(x;y)dy$.

117. Sketch the domain of integration and change the order of integration for $\int\limits_{-2}^2 dy \int\limits_{y^2-2}^{\frac{y^2}{2}} f(x;y)dx$.

118. Sketch the domain of integration and change the order of integration for $\int\limits_{-1}^1 dx \int\limits_{x^2}^{x^2+2} f(x;y)dy$.

119. Changing the order of integration express the sum $\int\limits_0^1 dy \int\limits_{\sqrt{y}}^1 f(x;y)dx + \int\limits_{-1}^0 dy \int\limits_0^{y+1} f(x;y)dx$ by one double integral.

120. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D (x-2y)dxdy$, if $D$ is the region given by inequalities $-1 \le x \le 2$ and $0 \le y \le x^2+1$.

121. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D (x^2 + y^2)dxdy$, if $D$ is bounded by the lines $y = x$, $x + y = 2a$ and $x = 0$.

122. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D xydxdy$, if $D$ is the least of segments bounded by the line $x + y = 2$ and circle $x^2 + y^2 = 2y$.

123. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D e^{x+y}dxdy$, if $D$ is the region bounded by $y = e^x$, $x = 0$ and $y = 2$.

124. Convert the double integral $\iint\limits_D f(x; y)dxdy$ to polar coordinates, if $D$ is the region determined by inequalities $1 \le x^2 + y^2 \le 4$ and $y \ge 0$.

125. Convert the double integral $\iint\limits_D f(x; y)dxdy$ to polar coordinates, if $D$ is bounded by the circles $x^2 + y^2 = 4x$ and $x^2 + y^2 = 8x$ and the lines $y = x$ and $y = 2x$.

126. Convert the double integral $\int\limits_{\frac{R}{2}}^{2R} dy \int\limits_{0}^{\sqrt{2Ry-y^2}} f(x; y)dx$ to polar coordinates.

127. Evaluate the double integral by converting it into polar coordinates $\int\limits_0^a dx \int\limits_0^{\sqrt{a^2-x^2}} e^{x^2+y^2}dy$.

128. Evaluate the double integral by converting it into polar coordinates $\iint\limits_D \frac{dxdy}{\sqrt{4 - x^2 - y^2}}$, if $D$ is the region determined by inequalities $x^2 + y^2 \le 4$, $x \ge 0$, $y \ge 0$.

129. Evaluate the double integral by converting it into polar coordinates $\int_0^R dx \int_0^{\sqrt{R^2-x^2}} \ln\left(1+x^2+y^2\right) dy$.

130. Evaluate the double integral by converting it into polar coordinates $\iint_D \sqrt{R^2-x^2-y^2}dxdy$, if $D$ is the circle $x^2+y^2 \le Rx$.

131. Evaluate the double integral by converting it into polar coordinates $\iint_D x\sqrt{x^2+y^2}dxdy$, if $D$ is bounded by the part of lemniscate $(x^2+y^2)^2 = a^2(x^2-y^2)$ where $x \ge 0$.

132. Evaluate the triple integral $\int_0^a dx \int_0^x dy \int_0^{xy} x^3y^2zdz$.

133. Evaluate the triple integral $\iiint_V \frac{dxdydz}{(x+y+z+1)^3}$, if $V$ is the region bounded by planes $x=0$, $y=0$, $z=0$ and $x+y+z=1$.

134. Evaluate the triple integral $\iiint_V xyzdxdydz$, if $V$ is bounded by the surfaces $y=x^2$, $x=y^2$, $z=xy$ and $z=0$.

135. Convert the triple integral $\iiint_V f(x;y;z)dxdydz$ into cylindrical coordinates, if $V$ is the region bounded by the planes $x=0$, $y=0$, $z=0$ and cylinders $x^2+y^2=4$ and $z=x^2+y^2$.

136. Evaluate the triple integral by converting it into cylindrical coordinates

$$\int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \int_{x^2+y^2}^{1} \frac{dz}{\sqrt{x^2+y^2}}$$

137. Evaluate the triple integral by converting it into cylindrical coordinates $\iiint_V z\sqrt{x^2+y^2}dxdydz$, if $V$ is the region determined by the inequalities $0 \le x \le 2$, $0 \le z \le 3$ and $0 \le y \le \sqrt{2x-x^2}$.

138. Convert the triple integral $\iiint\limits_V f(x; y; z)dxdydz$ into spherical coordinates, if $V$ is the region determined by the inequalities $1 \leq x^2 + y^2 + z^2 \leq 4$, $z \geq 0$ and $y \leq 0$.

139. Evaluate the triple integral by converting it into spherical coordinates

$$\int\limits_{-R}^{R} dx \int\limits_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} dy \int\limits_{0}^{\sqrt{R^2-x^2-y^2}} \frac{dz}{\sqrt{z}}$$

140. Evaluate the triple integral by converting it into spherical coordinates $\iiint\limits_V \sqrt{x^2 + y^2 + z^2}dxdydz$, if $V$ is the region determined by the inequalities $0 \leq y \leq 1$, $0 \leq x \leq \sqrt{1 - y^2}$ and $0 \leq z \leq \sqrt{1 - x^2 - y^2}$.

141. Compute the area bounded by $xy = 4$ and $x + y = 5$.

142. Compute the area bounded by$y = \dfrac{8a^3}{x^2 + 4a^2}$, $x = 2y$ and $x = 0$ provided $a$ is a positive constant.

143. Compute the volume of solid bounded by the planes $z = 0$, $y = 0$, $y = x$ and $x = 2$ and paraboloid of revolution $z = x^2 + y^2$.

144. Compute the volume of solid bounded by the hyperbolic paraboloid (saddle surface) $z = x^2 - y^2$ and the planes $z = 0$ and $x = 3$.

145. Compute the volume of solid bounded by the surfaces $z = x^2 + y^2$, $z = 2(x^2 + y^2)$, $y = x$ and $y^2 = x$.

146. Compute the volume of solid bounded by the sphere $x^2 + y^2 + z^2 = 4$ and paraboloid of revolution $3z = x^2 + y^2$.

147. Compute the volume of solid determined by the equations $y \geq 0$, $y \leq x$, $1 \leq x^2 + y^2 \leq 4$ and $0 \leq z \leq x^2 + y^2 + 1$.

148. Compute the volume of solid determined by the equations $x^2+y^2+z^2 \leq R^2$ and $x^2 + y^2 + z^2 \leq 2Rz$.

149. Compute the line integral $\int\limits_L (x^2 + y^2)ds$ where $L$ is the line segment from $A(1; 1)$ to $B(4; 4)$.

150. Compute the line integral $\int\limits_L y^2 ds$ where $L$ is the arc of cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ between the points $O(0;0)$ and $C(2a\pi;0)$.

151. Compute the line integral $\int\limits_L (x^2 + y^2 + z)ds$ where $L$ is the arc of helix $x = a\cos t$, $y = a\sin t$, $z = bt$ from $t = 0$ to $t = 2\pi$

152. Compute the line integral $\int\limits_L xyzds$ where $L$ is the quarter of circle $x = \dfrac{R}{2}\cos t$, $y = \dfrac{R}{2}\sin t$, $z = \dfrac{R\sqrt{3}}{2}$, which lies in the first octant.

153. Compute the line integral $\int\limits_L \dfrac{ydx + xdy}{x^2 + y^2}$ where $L$ is the segment of the line $y = x$ from $(1;1)$ to $(2;2)$.

154. Compute the line integral $\int\limits_L \arctan\dfrac{y}{x}dy - dx$ where $L$ is the arc of parabola $y = x^2$ from $O(0;0)$ to $A(1;1)$.

155. Compute the line integral $\int\limits_{AB} (x + y)dx + (x - y)dy$ where $AB$ is the arc of ellipse $x = a\cos t$, $y = b\sin t$ from $A(a;0)$ to $B(0;b)$.

156. Compute the line integral $\int\limits_L xdy - ydx$ where $L$ is the arc of astroid $x = a\cos^3 t$, $y = a\sin^3 t$ from $t = 0$ to $t = \dfrac{\pi}{2}$.

157. Compute the line integral $\int\limits_L \dfrac{x}{y}dx + \dfrac{dy}{y - 1}$ where $L$ is the arc of cycloid $x = t - \sin t$, $y = 1 - \cos t$ from $t = \dfrac{\pi}{6}$ to $t = \dfrac{\pi}{3}$.

158. Compute the line integral $\int\limits_{AB} \dfrac{xdx + ydy + zdz}{\sqrt{x^2 + y^2 + z^2 - x - y + 2z}}$ where $AB$ is the line segment from $A(1;1;1)$ to $B(4;4;4)$.

159. Compute the line integral $\int\limits_L yzdx + xzdy + xydz$ where $L$ is the arc of helix $x = a\cos t$, $y = a\sin t$, $z = bt$ from $t = 0$ to $t = 2\pi$.

160. Convert the line integral $\oint\limits_L (1 - x^3)ydx + x(1 + y^3)dy$ to the double integral over the region $D$ where $L$ is positively oriented, smooth, closed curve and $D$ the region enclosed by $L$.

161. Convert the line integral $\oint\limits_L e^x(1 - \cos y)dx + e^x(\sin y + y)dy$ to the double integral over the region $D$ where $L$ is positively oriented, smooth, closed curve and $D$ the region enclosed by $L$.

162. Use Green's theorem to find $\oint\limits_L (x + y^2)dx + (x + y)^2 dy$ where $L$ is the contour of triangle $ABC$ with vertices $A(1; 0)$, $B(1; 1)$ and $C(0; 1)$ with positive orientation.

163. Use Green's theorem to find $\oint\limits_L (5x - 3y)dx + (x - 4y)dy$ where $L$ is the circle $x^2 + y^2 = 1$ with positive orientation.

164. Use Green's theorem to find $\oint\limits_L 2xydx + x^2 dy$ where $L$ is the contour of square $|x| + |y| = 1$ with positive orientation.

165. Use Green's theorem to find $\oint\limits_L xy^2 dy - x^2 ydx$ where $L$ is the circle $x^2 + y^2 = 5$ with positive orientation.

166. Find the function $u$, if the total differential is $du = x^2 dx + y^2 dy$.

167. Find the function $u$, if the total differential is $du = (\cos y - 2xe^y)dx - (x^2 e^y + x\sin y)dy$.

168. Evaluate $\int\limits_{(0;0)}^{(2;1)} 2xydx + x^2 dy$.

169. Evaluate $\int\limits_{(-1;2)}^{(2;3)} ydx + xdy.$

170. Evaluate $\int\limits_{(1;1)}^{(2;2)} \frac{xdx + ydy}{\sqrt{x^2 + y^2}}.$

171. Evaluate $\iint\limits_S (x+y+z)d\sigma$ where $S$ is the part of the plain $\frac{x}{4}+\frac{y}{2}+z = 1$ in the first octant.

172. Evaluate $\iint\limits_S (x^2 + y^2)d\sigma$ where $S$ is the surface cut from the cone $z = \sqrt{x^2 + y^2}$ by the cylinder $x^2 + y^2 = 1$.

173. Evaluate $\iint\limits_S \sqrt{R^2 - x^2 - y^2}d\sigma$ where $S$ is the upper half of the sphere $z = \sqrt{R^2 - x^2 - y^2}$.

174. Evaluate $\iint\limits_S \sqrt{1 + x^2 + y^2}d\sigma$ where $S$ is the part of the saddle surface $z = xy$ cut by the cylinder $x^2 + y^2 = 1$.

175. Evaluate $\iint\limits_S xdydz + ydxdz + zdxdy$ where $S$ is that part of the plane $x + y + z = 1$ which is in the first octant. Choose the side where the normal forms the acute angles with coordinate axes.

176. Evaluate $\iint\limits_S x^2y^2zdxdy$ where $S$ is the upper side of the hemisphere $z = \sqrt{R^2 - x^2 - y^2}$.

177. Evaluate $\iint\limits_S xyzdzdy$ where $S$ is the lower side of the hemisphere $z = \sqrt{R^2 - x^2 - y^2}$.

178. Evaluate $\iint\limits_S xzdxdy + xydydz + yzdxdz+$ where $S$ is the inner side

of the pyramid determined by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

179. Write the general term of the series $\frac{1}{2} + \left(\frac{2}{5}\right)^3 + \left(\frac{3}{8}\right)^5 + \ldots$.

180. Write the general term of the series $1 - \frac{2}{7} + \frac{3}{13} - \frac{4}{19} + \ldots$.

181. Using the identity $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$, find the $n$th partial sum and the sum of the series $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{k(k+1)} + \ldots$.

182. Find the $n$th partial sum and the sum of the series $\frac{1}{3\cdot 6} + \frac{1}{6\cdot 9} + \frac{1}{9\cdot 12} + \ldots + \frac{1}{3k(3k+3)} + \ldots$.

183. find the $n$th partial sum and the sum of the series $\sum_{k=1}^{\infty}(\sqrt{k+2} - 2\sqrt{k+1} + \sqrt{k})$.

184. Use the Comparison Test to determine whether the series $\sum_{k=1}^{\infty} \frac{2^k}{5+3^k}$ converges or diverges.

185. Use the Comparison Test to determine whether the series $\sum_{k=2}^{\infty} \frac{1}{(k^3-1)^{\frac{1}{3}}}$ converges or diverges.

186. Use the d'Alembert Test to determine whether the series $1 + \frac{3}{2!} + \frac{6}{3!} + \frac{12}{4!} + \ldots + \frac{3\cdot 2^{n-2}}{n!} + \ldots$ converges or diverges.

187. Use the d'Alembert Test to determine whether the series $\sum_{k=1}^{\infty} \frac{k^2}{k!}$ converges or diverges.

188. Use the d'Alembert Test to determine whether the series $\sum_{k=0}^{\infty} \frac{3^k}{(3k+1)!}$ converges or diverges.

189. Use the Cauchy Test to determine whether the series $\sum_{k=1}^{\infty} \arcsin^k \frac{2k-1}{4k+3}$ converges or diverges.

190. Use the Cauchy Test to determine whether the series $\sum_{k=1}^{\infty} \ln^k \frac{2k+3}{k+1}$ converges or diverges.

191. Use the Cauchy Test to determine whether the series $\sum_{k=1}^{\infty} 2^k \left(\frac{k+2}{k+1}\right)^{-k^2}$ converges or diverges.

192. Use the Integral Test to determine whether the series $\frac{1}{4} + \frac{1}{7} + \frac{1}{10} + \ldots + \frac{1}{3k+1} + \ldots$ converges or diverges.

193. Use the Integral Test to determine whether the series $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ converges or diverges.

194. Use the Leibnitz's Test to determine whether the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{6}} + \ldots$ converges or diverges.

195. Use the Leibnitz's Test to determine whether the series $\sum_{k=1}^{\infty} \frac{\cos k\pi}{k^3}$ converges or diverges.

196. Does the series

$$1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \ldots + (-1)^{n+1} \frac{1}{(2n-1)^2} + \ldots$$

converges conditionally or absolutely?

197. Does the series

$$\frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2^2} + \frac{1}{3} \cdot \frac{1}{2^3} - \ldots + (-1)^{n+1} \frac{1}{n} \frac{1}{(2)^n} + \ldots$$

converges conditionally or absolutely?

198. Does the series

$$\frac{1}{\ln 2} - \frac{1}{\ln 3} + \frac{1}{\ln 4} - \frac{1}{\ln 5} + \ldots + (-1)^n \frac{1}{\ln n} + \ldots$$

converges conditionally or absolutely?

In exercises 199. - 203. find the values of $x$ for which the functional series is convergent.

199. $1 + \frac{x}{2} + \frac{x^2}{4} + \ldots + \frac{x^n}{2^n} + \ldots.$

200. $x - \frac{x^2}{2^2} + \frac{x^3}{3^2} - \frac{x^4}{4^2} + \ldots + (-1)^{n+1}\frac{x^n}{n^2} + \ldots.$

201. $\sin x + 2\sin\frac{x}{3} + 4\sin\frac{x}{9} + \ldots + 2^n \sin\frac{x}{3^n} + \ldots.$

202. $\frac{x}{1+\sqrt{1}} + \frac{x^2}{2+\sqrt{2}} + \frac{x^3}{3+\sqrt{3}} + \ldots + \frac{x^n}{n+\sqrt{n}} + \ldots.$

203. $\sum_{k=0}^{\infty} \ln^k(ex).$

In exercises 204. - 206. determine whether the functional series can be majorized.

204. $1 + \frac{x}{1^2} + \frac{x^2}{2^2} + \ldots + \frac{x^n}{n^2} + \ldots \quad 0 \le x \le 1.$

205. $1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \ldots + \frac{x^n}{n} + \ldots \quad 0 \le x \le 1.$

206. $\frac{\sin x}{1^2} + \frac{\sin 2x}{2^2} + \frac{\sin 3x}{3^2} + \ldots + \frac{\sin nx}{n^2} + \ldots \quad 0 \le x \le 2\pi.$

In exercises 207. -210. find the radius of convergence and the domain of convergence.

207. $\sum_{k=1}^{\infty} \frac{x^2}{k(k+1)}.$

208. $\sum_{k=0}^{\infty} \frac{(kx)^k}{k!}.$

209. $\sum_{k=0}^{\infty} \frac{k(x-2)^k}{3^k}.$

210. $\sum_{k=0}^{\infty} \frac{2^k(x+3)^k}{k!}$.

In exercises 211. - 216. expand the function in powers of $x$ and determine the domain of convergence.

211. $f(x) = \frac{1}{10+x}$.

212. $f(x) = e^{-x}$.

213. $f(x) = \frac{1}{1+x^2}$.

214. $f(x) = \sinh x$.

215. $f(x) = \cos^2 x$.

216. $f(x) = \arctan x$ (Remark: integrate the result of the exercise 213. in limits from 0 to $x$).

In exercises 217. - 221. find the Fourier series expansion of the given $2\pi$-periodic function defined on a half-open interval.

217. $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \le x \le \pi \end{cases}$

218. $f(x) = x$, if $-\pi < x \le \pi$.

219. $f(x) = x^2$, if $-\pi < x \le \pi$.

220. $f(x) = \sin ax$, if $-\pi < x \le \pi$.

221. $f(x) = \frac{\pi - x}{2}$, if $0 < x \le 2\pi$.

# Answers

**9.** No because the first function is additionally determined, if $x \le 0$ and $\sin y \le 0$. **10.** No because the first function is additionally determined, if $x < 0$ and $y < 0$. **11.** 0; -1; does not exist. **12.** 2. **13.** 0. **14.** Does not exist. **15.** 0. **16.** Does not exist. **17.** 0. **18.** $2x\sqrt[3]{y} + \frac{1}{2\sqrt{x}\sqrt[4]{y}}$; $\frac{x^2}{3\sqrt[3]{y^2}} - \frac{\sqrt{x}}{4y\sqrt[4]{y}}$. **19.** $\frac{2}{y \sin \frac{2x}{y}}$; $-\frac{2x}{y^2 \sin \frac{2x}{y}}$. **20.** $-\frac{1}{y}e^{-\frac{x}{y}}$; $\frac{x}{y^2}e^{-\frac{x}{y}}$. **21.** $y \cos xy -$

$\frac{y}{x^2}\sin\frac{y}{x}$; $x\cos xy+\frac{1}{x}\sin\frac{y}{x}$. **22.** $\frac{1}{\sqrt{x^2+y^x}}$; $\frac{y}{(x+\sqrt{x^2+y^x})\sqrt{x^2+y^x}}$. **23.** $-\frac{y}{2\sqrt{x}(x+y^2)}$; $\frac{\sqrt{x}}{x+y^2}$. **24.** $y\ln(x+y)+\frac{xy}{x+y}$; $x\ln(x+y)+\frac{xy}{x+y}$. **25.** $\frac{y}{xy+\ln z}$; $\frac{x}{xy+\ln z}$; $\frac{1}{z(xy+\ln z}$). **26.** $\frac{2x}{\cos^2(x^2+y^3+z^4)}$; $\frac{3y^2}{\cos^2(x^2+y^3+z^4)}$; $\frac{4z^3}{\cos^2(x^2+y^3+z^4)}$. **27.** $y^z x^{y^z-1}$; $x^{y^z}\ln x\cdot zy^{z-1}$; $x^{y^z}\ln x{\cdot}y^z\ln y$. **28.** $2x\cdot\frac{1-x^2-y^2-\sqrt{x^2+y^2}}{\left(1+\sqrt{x^2+y^2}\right)^2}$; $2y\cdot\frac{1-x^2-y^2-\sqrt{x^2+y^2}}{\left(1+\sqrt{x^2+y^2}\right)^2}$. **29.** $\frac{2}{5}$; $\frac{1}{5}$. **30.** 1; -1. **31.** $\frac{3}{2}$. **34.** $dz=\frac{ydx-xdy}{|y|\sqrt{y^2-x^2}}$. **35.** $dz=\left(x^2\cos\frac{x}{y}\cos\frac{y}{x}+y^2\sin\frac{x}{y}\sin\frac{y}{x}\right)\frac{ydx+xdy}{x^2+y^2}$ **36.** $dz=\frac{ydx-xdy}{y^2\tan\frac{x}{y}}$. **37.** $dw=x^{yz}\left(\frac{yzdx}{x}+z\ln xdy+y\ln xdz\right)$. **38.** $\frac{1}{36}$. **39.** $\Delta z=\frac{19}{635}$; $dz=\frac{19}{600}$. **40.** $\Delta z\approx 0,3764$; $dz=0,35$. **41.** 259,84. **42.** $2\frac{53}{54}$. **43.** $\frac{\pi}{6}$. **44.** 0,006. **45.** $\frac{x(2x^2-y^2)}{y(x^2-2y^2)}$. **46.** $-\frac{1}{3}$. **47.** $\frac{3}{4e}$. **48.** 1. **49.** $\frac{2-x}{z+1}$; $\frac{2y}{z+1}$. **50.** $-\frac{2}{2+\pi}$; $-\frac{\pi}{2+\pi}$. **51.** $\frac{e}{2}$; $-\frac{1}{2}$. **52.** $dz=-\frac{1}{\sin 2z}(\sin 2xdx+\sin 2ydy)$. **53.** $\frac{1+\ln x}{x^2\ln^2 x+2x\ln x+2}$. **54.** $\frac{2}{x^2+4}$. **55.** $\frac{1}{\cos^2\left(3t+\frac{2}{t^2}-\sqrt{t}\right)}\left(3-\frac{4}{t^3}-\frac{1}{2\sqrt{t}}\right)$. **56.** $e^{2x}\sin x$. **57.** $\frac{x+u\cos x+ve^x}{\sqrt{x^2+u^2+v^2}}$. **58.** $\frac{t(2-t\cos t)|\sin t|}{\sin t\sqrt{\sin^2 t-t^4}}$. **59.** $\frac{u\cos^2 v-v^2\sin u\cos u}{\sqrt{u^2\cos^2 v+v^2\cos^2 u}}$; $\frac{v\cos^2 u-u^2\sin v\cos v}{\sqrt{u^2\cos^2 v+v^2\cos^2 u}}$. **60.** $\frac{2}{u}$; $\frac{4\sinh v\cosh v}{\sinh^2 v+\cosh^2 v}$. **61.** $\frac{y(2x-y)}{1+x^2y^2(x-y)^2}$; $\frac{x(x-2y)}{1+x^2y^2(x-y)^2}$. **63.** $\frac{xy^3}{(1-x^2y^2)\sqrt{1-x^2y^2}}$; $\frac{x^3y}{(1-x^2y^2)\sqrt{1-x^2y^2}}$. $\frac{1}{(1-x^2y^2)\sqrt{1-x^2y^2}}$; **64.** $e^{xyz}(1+3xyz+x^2y^2z^2)$. **65.** 0; $\frac{1}{4}$; $-\frac{3}{8}$. **66.** $-\frac{4}{25}$; $\frac{3}{25}$; $\frac{4}{25}$. **67.**

0. **70.** $\dfrac{x-1}{1}=\dfrac{y+1}{2}=\dfrac{z-3}{0}$. **71.** $\dfrac{x-\sqrt{2}}{-\sqrt{2}}=\dfrac{y-\sqrt{2}}{\sqrt{2}}=\dfrac{z-\pi}{4}$. **72.** $\dfrac{x-\frac{\pi}{2}+1}{1}=\dfrac{y-1}{1}=\dfrac{z-2\sqrt{2}}{\sqrt{2}}$. **73.** $(-1;1;-1)$; $\left(-\frac{1}{3};\frac{1}{9};-\frac{1}{27}\right)$. **74.** $x+y-4z=\pi$. $\dfrac{x-2}{\frac{1}{4}}=\dfrac{y+2}{\frac{1}{4}}=\dfrac{z+\frac{\pi}{4}}{-1}$. **75.** $3x-4y-5z=0$; $\dfrac{x-3}{\frac{3}{5}}=\dfrac{y+4}{-\frac{4}{5}}=\dfrac{z-5}{-1}$. **76.** $z+1=0$; $\dfrac{x+1}{0}=\dfrac{y+\pi}{0}=\dfrac{z+1}{-1}$. **77.** $3x+4y+6z-22=0$; $\dfrac{x-2}{3}=\dfrac{y-1}{4}=\dfrac{z-2}{6}$. **79.** $\left(2;-2\frac{1}{4}\right)$. **80.** $\left(-\frac{1}{3};\frac{3}{4}\right)$; $\left(\frac{7}{3};-\frac{3}{4}\right)$. **81.** $\left(\frac{1}{2};\frac{1}{2};-\frac{1}{2}\right)$. **82.** $-\frac{15}{4}\sqrt{\frac{3}{13}}$. **83.** $-\frac{30}{13}$. **84.** $2+\sqrt{2}$. **85.** 0,4; **86.** $\frac{3\sqrt{5}}{4}$; **87.** $\frac{\sqrt{2}}{2}$. **88.** $(0;-1;0)$. **89.** $\operatorname{div}\overrightarrow{F}=\frac{1}{y}+\frac{1}{z}+\frac{1}{x}$; $\operatorname{rot}\overrightarrow{F}=\left(\frac{y}{z^2};\frac{z}{x^2};\frac{x}{y^2}\right)$. **90.** $\operatorname{div}\overrightarrow{F}=\dfrac{2x}{x^2-y^2}-\dfrac{1}{1+(z-y)^2}+xy$; $\operatorname{rot}\overrightarrow{F}=\left(xz-\dfrac{1}{1+(z-y)^2};-yz;\dfrac{2y}{x^2-y^2}\right)$. **91.** $\operatorname{div}\overrightarrow{F}=-\dfrac{3}{(x+y-z)^2}$; $\operatorname{rot}\overrightarrow{F}=\overrightarrow{\Theta}$. **92.** $\operatorname{div}\overrightarrow{F}=0$; $\operatorname{rot}\overrightarrow{F}=(2x;2x;2y-2z)$. **93.** Local minimum at $\left(-\frac{17}{143};\frac{7}{143}\right)$. **94.** Local maximum $z_{max}=6912$ at $(6;4)$. **95.** Local minimum $z_{min}=3\sqrt[3]{3}$ at $\left(\frac{1}{\sqrt[3]{3}};\frac{1}{\sqrt[3]{3}}\right)$. **96.** There is no local extremum at $(-2;0)$, local minimum at $(0;0)$. **97.** There is no local extremum at $(0;0)$, local minimum at $(1;1)$. **98.** $z_{min}=z(1;0)=-3$; $z_{max}=z(1;2)=17$. **99.** $z_{min}=z(0;2)=z(0;-2)=-4$; $z_{max}=z(2;0)=z(-2;0)=4$. **100.** $z_{min}=z(0;0)=0$; $z_{max}=z\left(\frac{\pi}{3};\frac{\pi}{3}\right)=\frac{3\sqrt{3}}{2}$. **101.** $z(1;1)=2$. **102.** $\left(-\frac{1}{2}\arctan\frac{b}{a};\frac{\pi}{4}-\frac{1}{2}\arctan\frac{b}{a}\right)$. **103.** $(3;3;3)$; $(-1;1;1)$; $(1;-1;1)$; $(1;1;-1)$. **104.** $\frac{1}{3}$, $\frac{1}{3}$ and $\frac{1}{3}$ m. **105.**

$\left(\sqrt{\frac{23}{18}}; \frac{11}{6}\right); \left(-\sqrt{\frac{23}{18}}; \frac{11}{6}\right)$. **106.** $\left(\frac{21}{13}; 2; \frac{63}{26}\right)$. **107.** $\frac{8}{3}$. **108.** $\ln\frac{25}{24}$.

**109.** $3\frac{3}{4}$. **110.** $\frac{19}{12}$. **111.** $\frac{6}{35}$. **112.** $\int\limits_{-1}^{1} dx \int\limits_{0}^{1-x^2} f(x;y)dy$.

**113.** $\int\limits_{0}^{a} dy \int\limits_{y}^{y+2a} f(x;y)dx$. **114.** $\int\limits_{-1}^{1} dx \int\limits_{x^2}^{\frac{2}{1+x^2}} f(x;y)dy$. **115.** $\int\limits_{0}^{1} dy \int\limits_{y^2}^{\sqrt[3]{y}} f(x;y)dx$. **116.** $\int\limits_{-1}^{0} dy \int\limits_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x;y)dx + \int\limits_{0}^{1} dy \int\limits_{y-1}^{1} f(x;y)dx$.

**117.** $\int\limits_{-2}^{2} dx \int\limits_{-\sqrt{x+2}}^{\sqrt{x+2}} f(x;y)dy - \int\limits_{0}^{2} dx \int\limits_{-\sqrt{2x}}^{\sqrt{2x}} f(x;y)dy$.

**118.** $\int\limits_{0}^{1} dy \int\limits_{-\sqrt{y}}^{\sqrt{y}} f(x;y)dx + \int\limits_{1}^{3} dy \int\limits_{-1}^{1} f(x;y)dx - \int\limits_{2}^{3} dy \int\limits_{-\sqrt{y-2}}^{\sqrt{y-2}} f(x;y)dx$. **119.** $\int\limits_{0}^{1} dx \int\limits_{x-1}^{x^2} f(x;y)dy$. **120.** $-10\frac{7}{20}$. **121.** $\frac{4}{3}a^4$. **122.** $\frac{1}{4}$.

**123.** $e$. **124.** $\int\limits_{0}^{\pi} d\varphi \int\limits_{1}^{2} f(\rho\cos\varphi; \rho\sin\varphi)\rho d\rho$. **125.** $\int\limits_{\frac{\pi}{4}}^{\arctan 2} d\varphi \int\limits_{4\cos\varphi}^{8\cos\varphi} f(\rho\cos\varphi; \rho\sin\varphi)\rho d\rho$.

**126.** $\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} d\varphi \int\limits_{\frac{R}{2\sin\varphi}}^{2R\sin\varphi} f(\rho\cos\varphi; \rho\sin\varphi)\rho d\rho$. **127.** $\frac{\pi}{4}(e^{a^2} - 1)$. **128.** $\pi$. **129.** $\frac{\pi}{4}\left[(1+R^2)\ln(1+R^2) - R^2\right]$. **130.** $\frac{R^3}{3}\left(\pi - \frac{4}{3}\right)$.

**131.** $\frac{2\sqrt{2}}{15}a^4$. **132.** $\frac{a^{11}}{110}$. **133.** $\frac{1}{2}\ln 2 - \frac{5}{16}$. **134.** $\frac{1}{96}$.

**135.** $\int\limits_{0}^{\frac{\pi}{2}} d\varphi \int\limits_{0}^{2} d\rho \int\limits_{0}^{\rho^2} f(\rho\cos\varphi; \rho\sin\varphi; z)\rho dz$. **136.** $\frac{4}{3}\pi$. **137.** 8.

**138.** $\int\limits_{\pi}^{2\pi} d\varphi \int\limits_{0}^{\frac{\pi}{2}} d\theta \int\limits_{1}^{2} f(r\cos\varphi\sin\theta; r\sin\varphi\sin\theta; r\cos\theta)r^2\sin\theta dr$. **139.**

$\frac{8}{5}\pi R^2\sqrt{R}$. **140.** $\frac{\pi}{8}$; **141.** $\frac{1}{2}(15-16\ln 2)$. **142.** $a^2(\pi-1)$. **143.** $\frac{16}{3}$. **144.** 27. **145.** $\frac{3}{35}$. **146.** $\frac{19}{6}\pi$. **147.** $\frac{21\pi}{16}$. **148.** $\frac{5\pi R^3}{12}$. **149.** $42\sqrt{2}$. **150.** $\frac{256}{15}a^3$. **151.** $2\pi\sqrt{a^2+b^2}(a^2+\pi b)$. **152.** $\frac{R^4\sqrt{3}}{32}$. **153.** $\ln 2$. **154.** $\frac{\pi}{2}-2$. **155.** $-\frac{a^2+b^2}{2}$. **156.** $\frac{3\pi ab}{16}$. **157.** $\frac{\pi^2}{24}+\frac{1-\sqrt{3}}{2}-\frac{1}{2}\ln 3$. **158.** $3\sqrt{3}$; **159.** 0. **160.** $\iint\limits_D (x^3+y^3)dxdy$. **161.** $\iint\limits_D e^x y dxdy$. **162.** $\frac{2}{3}$. **163.** $4\pi$. **164.** 0. **165.** $\frac{25\pi}{2}$. **166.** $u(x,y)=\frac{1}{3}(x^3+y^3)+C$. **167.** $u(x,y)=x\cos y-x^2e^y+C$. **168.** 4. **169.** 8. **170.** $\sqrt{2}$. **171.** $\frac{7\sqrt{21}}{3}$. **172.** $\frac{\pi\sqrt{2}}{2}$. **173.** $\pi R^3$. **174.** $\frac{3\pi}{2}$. **175.** $\frac{1}{2}$. **176.** $\frac{2\pi R^7}{105}$. **177.** 0. **178.** $-\frac{1}{8}$. **179.** $\left(\frac{k}{3k-1}\right)^{2k-1}$. **180.** $(-1)^k\frac{k}{6k-5}$. **181.** $S_n=1-\frac{1}{n+1}$; $S=1$. **182.** $S_n=\frac{1}{9}-\frac{1}{9(n+1)}$; $S=\frac{1}{9}$. **183**. $1-\sqrt{2}+\sqrt{n+2}-\sqrt{n+1}$; $1-\sqrt{2}$. **184.** Converges. **185.** Diverges. **186.** Converges. **187.** Converges. **188.** Converges. **189.** Converges. **190.** Converges. **191.** Converges. **192.** Diverges. **193.** Converges. **194.** Converges. **195.** Converges. **196.** Converges absolutely. **197.** Converges absolutely. **198.** Converges conditionally. **199.** $-2<x<2$. **200.** $-1\le x\le 1$. **201.** $-\infty<x<\infty$. **202.** $-1\le x<1$. **203.** $(e^{-2};1)$. **204.** Majorized. **205.** Not majorized. **206.** Majorized. **207.** 1; $[-1;1]$. **208.** $e^{-1}$; $[-e^{-1};e^{-1})$. **209.** 3; $(-1;5)$. **210.** $\infty$; $\mathbb{R}$. **211.** $\frac{1}{10}-\frac{x}{100}+\frac{x^2}{10^3}-\frac{x^3}{10^4}+\ldots$, converges for $-10<x<10$. **212.** $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots$, converges for $-\infty<x<\infty$. **213.** $1-x^2+x^4-x^6+\ldots$, converges for $-1<x<1$. **214.** $x+\frac{x^3}{3!}+\frac{x^5}{5!}+\ldots$, converges for $-\infty<x<\infty$. **215.** $1+\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^n(2x)^{2n}}{(2n)!}$, converges for $-\infty < x < \infty$. **216.** $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$, converges for $-1 \le x \le 1$. **217.** $\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2k+1)x}{2k+1}$. **218.** $2\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\sin kx$. **219.** $\frac{\pi^2}{3} + 4\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\cos kx$. **220.** $\frac{2\sin a\pi}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^k k}{a^2-k^2}\sin kx$. **221.** $\sum_{k=1}^{\infty}\frac{\sin kx}{k}$.