Exercises of Mathematical analysis II

In exercises 1. - 8. represent the domain of the function by the inequalities and make a sketch showing the domain in xy-plane.

1. $z = \sqrt{x - \sqrt{y}}$.

2. $z = \arcsin \dfrac{y+2}{x-1} + \ln y$.

3. $z = \sqrt{\sin \pi(x^2 + y^2)}$.

4. $z = \ln x - \ln \sin y$.

5. $z = \sqrt{4 - x^2} + \ln(y^2 - 4)$

6. $z = \sqrt{\arcsin \dfrac{x}{y}}$

7. $z = (y + \sqrt{y})\sqrt{\cos x}$

8. $z = \sqrt{x^2 + y^2 - 1} - 2\ln(9 - x^2 - y^2)$

9. Are the functions
$$z = \sqrt{x \sin y} \quad \text{and} \quad z = \sqrt{x}\sqrt{\sin y}$$
identical? Why?

10. Are the functions
$$z = \ln xy \quad \text{and} \quad z = \ln x + \ln y$$
identical? Why?

Evaluate the limit

11. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2 - y}{2x + y}$

12. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$

13. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sqrt{x^2y^2 + 1} - 1}{x^2 + y^2}$

14. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$

15. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy(x^2-y^2)}{x^2+y^2}$

16. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$

17. $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^3+y^3)}{x^2+y^2}$

In exercises 18. - 28. find partial derivatives with respect to every independent variable.

18. $z = x^2\sqrt[3]{y} + \dfrac{\sqrt{x}}{\sqrt[4]{y}}$

19. $z = \ln\tan\dfrac{x}{y}$

20. $z = e^{-\frac{x}{y}}$

21. $z = \sin xy - \cos\dfrac{y}{x}$

22. $z = \ln(x + \sqrt{x^2+y^2})$

23. $z = \arctan\dfrac{y}{\sqrt{x}}$

24. $z = xy\ln(x+y)$

25. $w = \ln(xy + \ln z)$

26. $w = \tan(x^2+y^3+z^4)$

27. $w = x^{y^z}$

28. $w = e^{2x}\cos(yz)$

29. Evaluate the partial derivatives of $z = \dfrac{x}{\sqrt{x^2+y^2}}$ at $(1;-2)$

30. $z = \dfrac{x\cos y - y\cos x}{1+\sin x+\sin y}$. Evaluate $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, if $x = y = 0$.

31. $w = \ln(1 + x + y^2 + z^3)$. Evaluate $\dfrac{\partial w}{\partial x} + \dfrac{\partial w}{\partial y} + \dfrac{\partial w}{\partial z}$ at the point $x = y = z = 1$

32. $z = \ln(x^2 - y^2)$; prove that $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} - \dfrac{2}{x+y} = 0$

33. For the function $z = xy + x \arctan \dfrac{y}{x}$ prove that et $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = xy + z$.

34. Find the total differential of the function $z = \arcsin \dfrac{x}{y}$.

35. Find the total differential of the function $z = \sin \dfrac{x}{y} \cos \dfrac{y}{x}$.

36. Find the total differential of the function $z = \ln \sin \dfrac{x}{y}$.

37. Find the total differential of the function $w = x^{yz}$.

38. Find the total differential of the function $z = \dfrac{xy}{x^2 - y^2}$, if $x = 2$, $y = 1$, $\Delta x = 0,01$ and $\Delta y = 0,03$.

39. Evaluate the increment Δz and total differential dz of the function $z = \dfrac{x+y}{x-y}$, if $x = -3$, $y = 7$, $\Delta x = -\dfrac{1}{3}$ and $\Delta y = \dfrac{1}{4}$.

40. Evaluate the increment Δz and total differential dz of the function $z = xy + \dfrac{x}{y}$, if x changes from -1 to $-0,8$ and y from 2 to $2,2$.

41. Using total differential, compute the approximate value of $1,96^3 \cdot 2,03^5$.

42. Using total differential, compute the approximate value of $\dfrac{\sqrt{82}}{\sqrt[3]{28}}$.

43. Using total differential, compute the approximate value of $\arcsin \dfrac{\sqrt{1,04}}{2,04}$.

44. Using total differential, compute the approximate value of $\ln\left(\sqrt[5]{0,98} + \sqrt[4]{1,04} - 1\right)$.

45. Find $\dfrac{dy}{dx}$, if $x^2y^2 - x^4 - y^4 = a^2$.

46. Find $\dfrac{dy}{dx}$, if $2y^3 + 3x^2y + \ln x = 0$ and evaluate it at $x = 1$.

47. Find $\dfrac{dy}{dx}$, if $y=\sqrt{x}\ln\dfrac{x}{y}$ and evaluate it at the point $(e^2;e)$.

48. Find $\dfrac{dy}{dx}$, if $x^y=y^x$ and evaluate it at the point $(1;1)$.

49. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, if $x^2-2y^2+z^2-4x+2z-5=0$.

50. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, if $z=\cos xy-\sin xz$ and evaluate these at the point $\left(\dfrac{\pi}{2};1;0\right)$.

51. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, if $xyz=e^z$ and evaluate these at the point $(e^{-1};-1;-1)$.

52. Find $\dfrac{dz}{dx}$, if $z=\arctan(xy+1)$ and $y=\ln x$.

53. Find $\dfrac{dz}{dx}$, if $z=\arcsin\dfrac{x}{y}$ and $y=\sqrt{x^2+4}$.

54. Find $\dfrac{dz}{dt}$, if $z=\tan(3t+2x^2-y)$, $x=\dfrac{1}{t}$ and $y=\sqrt{t}$.

55. Find $\dfrac{du}{dx}$, if $u=\dfrac{e^{2x}}{5}(y-z)$, $y=2\sin x$ and $z=\cos x$.

56. Find $\dfrac{dw}{dx}$, if $w=\sqrt{x^2+u^2+v^2}$, $u=\sin x$ and $v=e^x$.

57. Find $\dfrac{dz}{dt}$, if $z=\arcsin\dfrac{y}{x}$, $x=\sin t$ and $y=t^2$.

58. Find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$, if $z=\sqrt{x^2+y^2}$, $x=u\cos v$ and $y=v\cos u$.

59. Find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$, if $z=\ln(x^2+y^2)$, $x=u\cosh v$ and $y=v\sinh u$.

60. Find the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ of the function $z=\arctan uv$, if $u=xy$ and $v=x-y$.

61. Prove that $\dfrac{\partial^2 z}{\partial x\partial y}=\dfrac{\partial^2 z}{\partial y\partial x}$, if $z=e^x(\cos y+x\sin y)$.

62. Find $\dfrac{\partial^2 z}{\partial x^2}$, $\dfrac{\partial^2 z}{\partial y^2}$ and $\dfrac{\partial^2 z}{\partial x \partial y}$, if $z = \arcsin(xy)$.

63. Find $\dfrac{\partial^3 w}{\partial x \partial y \partial z}$, if $w = e^{xyz}$.

64. Evaluate all second order derivatives of the function $z = \dfrac{x}{y^2}$ at the point $(-1; -2)$.

65. Evaluate all second order derivatives of the function $z = \arcsin \dfrac{x}{\sqrt{x^2 + y^2}}$ at the point $(1; -2)$.

66. Evaluate
$$\frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y} \right)^2$$
at the point $(0; -2)$, if $z = \dfrac{\cos x^2}{y}$.

67. For the function $z = \ln\left(e^x + e^y\right)$ prove that $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 1$ and $\dfrac{\partial^2 z}{\partial x^2} \dfrac{\partial^2 z}{\partial y^2} - \left(\dfrac{\partial^2 z}{\partial x \partial y} \right)^2 = 0$.

68. Prove that the function $z = \dfrac{x^2 y^2}{x + y}$ satisfies the equation
$$x \frac{\partial^2 z}{\partial x^2} + y \frac{\partial^2 z}{\partial x \partial y} = 2 \frac{\partial z}{\partial x}$$

69. Find the gradient vector for the scalar field $z = x - 3y + \sqrt{3xy}$ at the point $(3; 4)$.

70. Find the points at which the gradient vector of the scalar field $z = \ln\left(x + \dfrac{1}{y} \right)$ is $\overrightarrow{a} = \left(1; -\dfrac{16}{9} \right)$.

71. Find the gradient vector for the scalar field $w = \arcsin \dfrac{\sqrt{x^2 + y^2}}{z}$ at the point $(1; 1; 2)$.

72. Find the directional derivative of the function $z = \arctan \dfrac{y}{x} - \dfrac{4y}{x}$ at the point $(1; \sqrt{3})$ in direction the point $(2; 3\sqrt{3})$.

73. Find the directional derivative of the function $w = xyz$ at the point $A(-2;1;3)$ in the direction of $\overrightarrow{s} = (4;3;12)$.

74. Find the directional derivative of the function $w = x^2y^2 - z^2 + 2xyz$ at the point $B(1;1;0)$ in direction forming with coordinate axes the angles $60°$, $45°$ and $60°$ respectively.

75. Find the greatest rate of change of the function $z = \ln(x^2 + y^2)$ at the point $C(-3;4)$

76. Find the greatest value of the derivative of function given by the equation $x^2 + y^3 - z^2 - 1 = 0$ at the point $(3;2;4)$.

77. Find the steepest ascent of the surface $z = \arctan \dfrac{y}{x}$ at the point $(1;1)$.

78. Find the direction of greatest increase of the function $f(x, y, z) = x \sin z - y \cos z$ at the origin.

79. Find the divergence and curl of the vector field $\overrightarrow{F} = \left(\dfrac{x}{y}; \dfrac{y}{z}; \dfrac{z}{x}\right)$.

80. Find the divergence and curl of the vector field $\overrightarrow{F} = (\ln(x^2 - y^2); \arctan(z - y); xyz)$.

81. Find the divergence and curl of the vector field $\overrightarrow{F} = \operatorname{grad} w$, if $w = \ln(x + y - z)$.

82. Find the divergence and curl of the vector field $\overrightarrow{F} = \operatorname{rot} \overrightarrow{G}$, if $\overrightarrow{G} = (x^2y; y^2z; x^2z)$.

83. Find the local extrema of the function $z = 4x^2 - xy + 9y^2 + x - y$ and determine their type.

84. Find the local extremum points of the function $z = x^3y^2(12 - x - y)$, satisfying the conditions $x > 0$ and $y > 0$ and determine their type.

85. Find the local extrema of the function $z = x^2 + xy + y^2 + \dfrac{1}{x} + \dfrac{1}{y}$ and determine their type.

86. Find the local extrema of the function $z = e^x(x^2 + y^2)$ and determine their type.

87. Find the local extrema of the function $z = x^3 + y^3 - 3xy$ and determine their type.

88. Find the extremal values of the function $z = \dfrac{1}{x} + \dfrac{1}{y}$ under the condition $x + y = 2$.

89. Find the extremal values of the function $z = a\cos^2 x + b\cos^2 y$ under the condition $y - x = \dfrac{\pi}{4}$.

90. Find the extremal values of the function $w = x + y + z$ under the condition $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 1$.

91. The sum of three edges of the rectangular box, passing one vertex is 1 m. Find the dimensions of this rectangular box so that the volume is the greatest.

92. Evaluate the double integral $\displaystyle\iint_D (x^2 + y^2)dxdy$, if D is the quadrate $0 \le x \le 1$ and $1 \le y \le 2$.

93. Evaluate the double integral $\displaystyle\iint_D \frac{dxdy}{(x+y)^2}$, if D is the quadrate $1 \le x \le 2$ and $3 \le y \le 4$.

94. Evaluate the double integral $\displaystyle\int_1^2 dx \int_x^{x\sqrt{3}} xydy$.

95. Evaluate the double integral $\displaystyle\int_0^1 dx \int_{-x}^{x+1} (xy + y)dy$.

96. Evaluate the double integral $\displaystyle\int_0^1 dx \int_{x^2}^{\sqrt{x}} (x^2 + y^2)dy$.

97. Sketch the domain of integration and determine the limits of integration for $\displaystyle\iint_D f(x;y)dxdy$, if D is the region bounded by the line $y = 0$ and the parabola $y = 1 - x^2$.

98. Sketch the domain of integration and determine the limits of integration for $\displaystyle\iint_D f(x;y)dxdy$, if D is the parallelogram bounded by the lines $y = 0$, $y = a$, $y = x$ and $y = x - 2a$.

99. Sketch the domain of integration and determine the limits of integration for $\iint\limits_D f(x;y)dxdy$, if D is the region bounded by $y = \dfrac{2}{1+x^2}$ and $y = x^2$.

100. Sketch the domain of integration and change the order of integration for $\int\limits_0^1 dx \int\limits_{x^3}^{\sqrt{x}} f(x;y)dy$.

101. Sketch the domain of integration and change the order of integration for $\int\limits_{-1}^1 dx \int\limits_{-\sqrt{1-x^2}}^{x+1} f(x;y)dy$.

102. Sketch the domain of integration and change the order of integration for $\int\limits_{-2}^2 dy \int\limits_{y^2-2}^{\frac{y^2}{2}} f(x;y)dx$.

103. Sketch the domain of integration and change the order of integration for $\int\limits_{-1}^1 dx \int\limits_{x^2}^{x^2+2} f(x;y)dy$.

104. Changing the order of integration express the sum $\int\limits_0^1 dy \int\limits_{\sqrt{y}}^1 f(x;y)dx +$ $\int\limits_{-1}^0 dy \int\limits_0^{y+1} f(x;y)dx$ by one double integral.

105. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D (x-2y)dxdy$, if D is the region given by inequalities $-1 \le x \le 2$ and $0 \le y \le x^2+1$.

106. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D (x^2+y^2)dxdy$, if D is bounded by the lines $y = x$, $x + y = 2a$ and $x = 0$.

107. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D xydxdy$, if D is the least of segments bounded by the line $x + y = 2$ and circle $x^2 + y^2 = 2y$.

108. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D e^{x+y}dxdy$, if D is the region bounded by $y = e^x$, $x = 0$ and $y = 2$.

109. Convert the double integral $\iint\limits_D f(x; y)dxdy$ to polar coordinates, if D is the region determined by inequalities $1 \leq x^2 + y^2 \leq 4$ and $y \geq 0$.

110. Convert the double integral $\iint\limits_D f(x; y)dxdy$ to polar coordinates, if D is bounded by the circles $x^2 + y^2 = 4x$ and $x^2 + y^2 = 8x$ and the lines $y = x$ and $y = 2x$.

111. Convert the double integral $\int\limits_{\frac{R}{2}}^{2R} dy \int\limits_0^{\sqrt{2Ry-y^2}} f(x; y)dx$ to polar coordinates.

112. Evaluate the double integral by converting it into polar coordinates $\int\limits_0^a dx \int\limits_0^{\sqrt{a^2-x^2}} e^{x^2+y^2}dy$.

113. Evaluate the double integral by converting it into polar coordinates $\iint\limits_D \frac{dxdy}{\sqrt{4 - x^2 - y^2}}$, if D is the region determined by inequalities $x^2 + y^2 \leq 4$, $x \geq 0$, $y \geq 0$.

114. Evaluate the double integral by converting it into polar coordinates $\int\limits_0^R dx \int\limits_0^{\sqrt{R^2-x^2}} \ln\left(1 + x^2 + y^2\right) dy$.

115. Evaluate the double integral by converting it into polar coordinates $\iint\limits_D \sqrt{R^2 - x^2 - y^2}dxdy$, if D is the circle $x^2 + y^2 \leq Rx$.

116. Evaluate the double integral by converting it into polar coordinates $\iint\limits_D x\sqrt{x^2 + y^2}dxdy$, if D is bounded by the part of lemniscate $(x^2 + y^2)^2 = a^2(x^2 - y^2)$ where $x \geq 0$.

117. Evaluate the triple integral $\int\limits_0^a dx \int\limits_0^x dy \int\limits_0^{xy} x^3y^2zdz$.

118. Evaluate the triple integral $\iiint\limits_V \dfrac{dxdydz}{(x + y + z + 1)^3}$, if V is the region bounded by planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

119. Evaluate the triple integral $\iiint\limits_V xyzdxdydz$, if V is bounded by the surfaces $y = x^2$, $x = y^2$, $z = xy$ and $z = 0$.

120. Convert the triple integral $\iiint\limits_V f(x; y; z)dxdydz$ into cylindrical coordinates, if V is the region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and cylinders $x^2 + y^2 = 4$ and $z = x^2 + y^2$.

121. Evaluate the triple integral by converting it into cylindrical coordinates

$$\int\limits_{-1}^{1} dx \int\limits_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \int\limits_{x^2+y^2}^{1} \frac{dz}{\sqrt{x^2 + y^2}}$$

122. Evaluate the triple integral by converting it into cylindrical coordinates $\iiint\limits_V z\sqrt{x^2 + y^2}dxdydz$, if V is the region determined by the inequalities $0 \leq x \leq 2$, $0 \leq z \leq 3$ and $0 \leq y \leq \sqrt{2x - x^2}$.

123. Compute the area bounded by $xy = 4$ and $x + y = 5$.

124. Compute the area bounded by$y = \dfrac{8a^3}{x^2 + 4a^2}$, $x = 2y$ and $x = 0$ provided a is a positive constant.

125. Compute the volume of solid bounded by the planes $z = 0$, $y = 0$, $y = x$ and $x = 2$ and paraboloid of revolution $z = x^2 + y^2$.

126. Compute the volume of solid bounded by the hyperbolic paraboloid (saddle surface) $z = x^2 - y^2$ and the planes $z = 0$ and $x = 3$.

127. Compute the volume of solid bounded by the surfaces $z = x^2 + y^2$, $z = 2(x^2 + y^2)$, $y = x$ and $y^2 = x$.

128. Compute the volume of solid bounded by the sphere $x^2 + y^2 + z^2 = 4$ and paraboloid of revolution $3z = x^2 + y^2$.

129. Compute the volume of solid determined by inequalities $y \geq 0$, $y \leq x$, $1 \leq x^2 + y^2 \leq 4$ and $0 \leq z \leq x^2 + y^2 + 1$.

130. Compute the line integral $\int\limits_L (x^2 + y^2)ds$ where L is the line segment from $A(1;1)$ to $B(4;4)$.

131. Compute the line integral $\int\limits_L y^2 ds$ where L is the arc of cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ between the points $O(0;0)$ and $C(2a\pi;0)$.

132. Compute the line integral $\int\limits_L (x^2 + y^2 + z)ds$ where L is the arc of helix $x = a\cos t$, $y = a\sin t$, $z = bt$ from $t = 0$ to $t = 2\pi$

133. Compute the line integral $\int\limits_L xyzds$ where L is the quarter of circle $x = \frac{R}{2}\cos t$, $y = \frac{R}{2}\sin t$, $z = \frac{R\sqrt{3}}{2}$, which lies in the first octant.

134. Compute the line integral $\int\limits_L \frac{ydx + xdy}{x^2 + y^2}$ where L is the segment of the line $y = x$ from $(1;1)$ to $(2;2)$.

135. Compute the line integral $\int\limits_L \arctan\frac{y}{x}dy - dx$ where L is the arc of parabola $y = x^2$ from $O(0;0)$ to $A(1;1)$.

136. Compute the line integral $\int\limits_{AB} (x+y)dx+(x-y)dy$ where AB is the arc of ellipse $x = a\cos t$, $y = b\sin t$ from $A(a;0)$ to $B(0;b)$.

137. Compute the line integral $\int\limits_{L} xdy - ydx$ where L is the arc of astroid $x = a\cos^3 t$, $y = a\sin^3 t$ from $t = 0$ to $t = \dfrac{\pi}{2}$.

138. Compute the line integral $\int\limits_{L} \dfrac{x}{y}dx + \dfrac{dy}{y-1}$ where L is the arc of cycloid $x = t - \sin t$, $y = 1 - \cos t$ from $t = \dfrac{\pi}{6}$ to $t = \dfrac{\pi}{3}$.

139. Compute the line integral $\int\limits_{AB} \dfrac{xdx + ydy + zdz}{\sqrt{x^2 + y^2 + z^2 - x - y + 2z}}$ where AB is the line segment from $A(1;1;1)$ to $B(4;4;4)$.

140. Compute the line integral $\int\limits_{L} yzdx + xzdy + xydz$ where L is the arc of helix $x = a\cos t$, $y = a\sin t$, $z = bt$ from $t = 0$ to $t = 2\pi$.

141. Convert the line integral $\oint\limits_{L} (1 - x^3)ydx + x(1 + y^3)dy$ to the double integral over the region D where L is positively oriented, smooth, closed curve and D the region enclosed by L.

142. Convert the line integral $\oint\limits_{L} e^x(1-\cos y)dx+e^x(\sin y+y)dy$ to the double integral over the region D where L is positively oriented, smooth, closed curve and D the region enclosed by L.

143. Use Green's theorem to find $\oint\limits_{L} (x + y^2)dx + (x + y)^2dy$ where L is the contour of triangle ABC with vertices $A(1;0)$, $B(1;1)$ and $C(0;1)$ with positive orientation.

144. Use Green's theorem to find $\oint\limits_{L} (5x - 3y)dx + (x - 4y)dy$ where L is the circle $x^2 + y^2 = 1$ with positive orientation.

145. Use Green's theorem to find $\oint\limits_L 2xydx + x^2dy$ where L is the contour of square $|x| + |y| = 1$ with positive orientation.

146. Use Green's theorem to find $\oint\limits_L xy^2dy - x^2ydx$ where L is the circle $x^2 + y^2 = 5$ with positive orientation.

147. Evaluate $\int\limits_{(0;0)}^{(2;1)} 2xydx + x^2dy$

148. Evaluate $\int\limits_{(-1;2)}^{(2;3)} ydx + xdy$

149. Evaluate $\int\limits_{(1;1)}^{(2;2)} \frac{xdx + ydy}{\sqrt{x^2 + y^2}}$

150. Evaluate $\iint\limits_S (x+y+z)d\sigma$ where S is the part of the plain $\frac{x}{4}+\frac{y}{2}+z = 1$ in the first octant.

151. Evaluate $\iint\limits_S (x^2 + y^2)d\sigma$ where S is the surface cut from the cone $z = \sqrt{x^2 + y^2}$ by the cylinder $x^2 + y^2 = 1$.

152. Evaluate $\iint\limits_S \sqrt{R^2 - x^2 - y^2}d\sigma$ where S is the upper half of the sphere $z = \sqrt{R^2 - x^2 - y^2}$.

153. Evaluate $\iint\limits_S \sqrt{1 + x^2 + y^2}d\sigma$ where S is the part of the saddle surface $z = xy$ cut by the cylinder $x^2 + y^2 = 1$.

154. Evaluate $\iint\limits_S xdydz + ydxdz + zdxdy$ where S is that part of the plane $x + y + z = 1$ which is in the first octant. Choose the side where the normal forms the acute angles with coordinate axes.

155. Evaluate $\iint\limits_S x^2y^2z dx dy$ where S is the upper side of the hemisphere $z = \sqrt{R^2 - x^2 - y^2}$.

156. Evaluate $\iint\limits_S xyz dz dy$ where S is the lower side of the hemisphere $z = \sqrt{R^2 - x^2 - y^2}$.

157. Evaluate $\iint\limits_S xz dx dy + xy dy dz + yz dx dz +$ where S is the inner side of the pyramid determined by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

158. Write the general term of the series $\frac{1}{2} + \left(\frac{2}{5}\right)^3 + \left(\frac{3}{8}\right)^5 + \ldots$.

159. Write the general term of the series $1 - \frac{2}{7} + \frac{3}{13} - \frac{4}{19} + \ldots$.

160. Using the identity $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$, find the nth partial sum and the sum of the series $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{k(k+1)} + \ldots$.

161. Find the nth partial sum and the sum of the series $\frac{1}{3 \cdot 6} + \frac{1}{6 \cdot 9} + \frac{1}{9 \cdot 12} + \ldots + \frac{1}{3k(3k+3)} + \ldots$.

162. find the nth partial sum and the sum of the series $\sum_{k=1}^{\infty}(\sqrt{k+2} - 2\sqrt{k+1} + \sqrt{k})$.

163. Use the Comparison Test to determine whether the series $\sum_{k=1}^{\infty} \frac{2^k}{5 + 3^k}$ converges or diverges.

164. Use the Comparison Test to determine whether the series $\sum_{k=2}^{\infty} \frac{1}{(k^3 - 1)^{\frac{1}{3}}}$ converges or diverges.

165. Use the d'Alembert Test to determine whether the series $1+\frac{3}{2!}+\frac{6}{3!}+\frac{12}{4!}+\ldots+\frac{3\cdot 2^{n-2}}{n!}+\ldots$ converges or diverges.

166. Use the d'Alembert Test to determine whether the series $\sum_{k=1}^{\infty}\frac{k^2}{k!}$ converges or diverges.

167. Use the d'Alembert Test to determine whether the series $\sum_{k=0}^{\infty}\frac{3^k}{(3k+1)!}$ converges or diverges.

168. Use the Cauchy Test to determine whether the series $\sum_{k=1}^{\infty}\arcsin^k\frac{2k-1}{4k+3}$ converges or diverges.

169. Use the Cauchy Test to determine whether the series $\sum_{k=1}^{\infty}\ln^k\frac{2k+3}{k+1}$ converges or diverges.

170. Use the Cauchy Test to determine whether the series $\sum_{k=1}^{\infty}2^k\left(\frac{k+2}{k+1}\right)^{-k^2}$ converges or diverges.

171. Use the Integral Test to determine whether the series $\frac{1}{4}+\frac{1}{7}+\frac{1}{10}+\ldots+\frac{1}{3k+1}+\ldots$ converges or diverges.

172. Use the Integral Test to determine whether the series $\sum_{k=2}^{\infty}\frac{1}{k(\ln k)^2}$ converges or diverges.

173. Use the Leibnitz's Test to determine whether the series $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{6}}+\ldots$ converges or diverges.

174. Use the Leibnitz's Test to determine whether the series $\sum_{k=1}^{\infty}\frac{\cos k\pi}{k^3}$ converges or diverges.

175. Does the series

$$1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \ldots + (-1)^{n+1}\frac{1}{(2n-1)^2} + \ldots$$

converges conditionally or absolutely?

176. Does the series

$$\frac{1}{2} - \frac{1}{2}\cdot\frac{1}{2^2} + \frac{1}{3}\cdot\frac{1}{2^3} - \ldots + (-1)^{n+1}\frac{1}{n}\frac{1}{(2)^n} + \ldots$$

converges conditionally or absolutely?

177. Does the series

$$\frac{1}{\ln 2} - \frac{1}{\ln 3} + \frac{1}{\ln 4} - \frac{1}{\ln 5} + \ldots + (-1)^n\frac{1}{\ln n} + \ldots$$

converges conditionally or absolutely?

In exercises 178. -181. find the radius of convergence and the domain of convergence.

178. $\displaystyle\sum_{k=1}^{\infty} \frac{x^k}{k(k+1)}$.

179. $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{\sqrt{k}}$.

180. $\displaystyle\sum_{k=0}^{\infty} \frac{k(x-2)^k}{3^k}$.

181. $\displaystyle\sum_{k=0}^{\infty} \frac{2^k(x+3)^k}{k!}$.

In exercises 182. - 187. expand the function in powers of x and determine the domain of convergence.

182. $f(x) = \dfrac{1}{10+x}$.

183. $f(x) = e^{-x}$.

184. $f(x) = \dfrac{1}{1+x^2}$.

185. $f(x) = \sinh x$.

186. $f(x) = \cos^2 x$.

187. $f(x) = \arctan x$ (Remark: integrate the result of the exercise 184. in limits from 0 to x).

In exercises 188. - 192. find the Fourier series expansion of the given 2π-periodic function defined on a half-open interval.

188. $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \leq x \leq \pi \end{cases}$

189. $f(x) = x$, if $-\pi < x \leq \pi$.

190. $f(x) = x^2$, if $-\pi < x \leq \pi$.

191. $f(x) = \sin ax$, if $-\pi < x \leq \pi$.

192. $f(x) = \dfrac{\pi - x}{2}$, if $0 < x \leq 2\pi$.

In exercises 193. - 196. determine the Fourier transform of function given.

193. Heaviside unit step function $H(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$

194. Rectangular pulse $f(x) = \begin{cases} A, & \text{if } |x| \leq 2 \\ 0, & \text{if } |x| > 2 \end{cases}$

195. Two sided exponential pulse

$$f(x) = \begin{cases} e^{ax}, & \text{if } x \leq 0 \\ e^{-ax}, & \text{if } x > 0 \end{cases} \quad \text{provided } (a > 0)$$

196. $f(x) = \begin{cases} \sin ax, & \text{if } |x| \leq \dfrac{\pi}{a} \\ 0, & \text{if } |x| > \dfrac{\pi}{a} \end{cases}$

Answers

9. No, because the first function is also defined if $x \leq 0$ and $\sin y \leq 0$. **10.** No, because the first function is also defined if $x < 0$ and $y < 0$. **11.** Does not exist. **12.** 2 **13.** 0. **14.** Does not exist. **15.** 0. **16.** Does not exist. **17.** 0 **18.**

$2x\sqrt[3]{y}+\dfrac{1}{2\sqrt{x}\sqrt[4]{y}}$; $\dfrac{x^2}{3\sqrt[3]{y^2}}-\dfrac{\sqrt{x}}{4y\sqrt[4]{y}}$ **19.** $\dfrac{2}{y\sin\dfrac{2x}{y}}$; $-\dfrac{2x}{y^2\sin\dfrac{2x}{y}}$ **20.** $-\dfrac{1}{y}e^{-\frac{x}{y}}$; $\dfrac{x}{y^2}e^{-\frac{x}{y}}$ **21.** $y\cos xy-\dfrac{y}{x^2}\sin\dfrac{y}{x}$; $x\cos xy+\dfrac{1}{x}\sin\dfrac{y}{x}$ **22.** $\dfrac{1}{\sqrt{x^2+y^x}}$; $\dfrac{y}{(x+\sqrt{x^2+y^x})\sqrt{x^2+y^x}}$. **23.** $-\dfrac{y}{2\sqrt{x}(x+y^2)}$; $\dfrac{\sqrt{x}}{x+y^2}$
24. $y\ln(x+y)+\dfrac{xy}{x+y}$; $x\ln(x+y)+\dfrac{xy}{x+y}$ **25.** $\dfrac{y}{xy+\ln z}$; $\dfrac{x}{xy+\ln z}$; $\dfrac{1}{z(xy+\ln z)}$ **26.** $\dfrac{2x}{\cos^2(x^2+y^3+z^4)}$; $\dfrac{3y^2}{\cos^2(x^2+y^3+z^4)}$; $\dfrac{4z^3}{\cos^2(x^2+y^3+z^4)}$
27. $y^zx^{y^z-1}$; $x^{y^z}\ln x\cdot zy^{z-1}$; $x^{y^z}\ln x\cdot y^z\ln y$. **28.** $2e^{2x}\cos(yz)$; $-ze^{2x}\sin(yz)$; $-ye^{2x}\sin(yz)$ **29.** $\dfrac{4}{5\sqrt{5}}$; $\dfrac{2}{5\sqrt{5}}$ **30.** 1; -1 **31.** $\dfrac{3}{2}$ **34.** $dz=\dfrac{ydx-xdy}{|y|\sqrt{y^2-x^2}}$. **35.** $dz=\left(x^2\cos\dfrac{x}{y}\cos\dfrac{y}{x}+y^2\sin\dfrac{x}{y}\sin\dfrac{y}{x}\right)\dfrac{ydx+xdy}{x^2+y^2}$
36. $dz=\dfrac{ydx-xdy}{y^2\tan\dfrac{x}{y}}$ **37.** $dw=x^{yz}\left(\dfrac{yzdx}{x}+z\ln xdy+y\ln xdz\right)$ **38.** $\dfrac{1}{36}$ **39.** $\Delta z=\dfrac{19}{635}$; $dz=\dfrac{19}{600}$ **40.** $\Delta z\approx 0,3764$; $dz=0,35$
41. 259,84. **42.** $2\dfrac{53}{54}$ **43.** $\dfrac{\pi}{6}$. **44.** 0,006. **45.** $\dfrac{x(2x^2-y^2)}{y(x^2-2y^2)}$ **46.** $-\dfrac{1}{3}$ **47.** $\dfrac{3}{4e}$ **48.** 1 **49.** $\dfrac{2-x}{z+1}$; $\dfrac{2y}{z+1}$
50. $-\dfrac{2}{2+\pi}$; $-\dfrac{\pi}{2+\pi}$ **51.** $\dfrac{e}{2}$; $-\dfrac{1}{2}$ **52.** $\dfrac{1+\ln x}{x^2\ln^2x+2x\ln x+2}$ **53.** $\dfrac{2}{x^2+4}$ **54.** $\dfrac{1}{\cos^2\left(3t+\frac{2}{t^2}-\sqrt{t}\right)}\left(3-\dfrac{4}{t^3}-\dfrac{1}{2\sqrt{t}}\right)$. **55.** $e^{2x}\sin x$
56. $\dfrac{x+u\cos x+ve^x}{\sqrt{x^2+u^2+v^2}}$ **57.** $\dfrac{t(2-t\cos t)|\sin t|}{\sin t\sqrt{\sin^2t-t^4}}$ **56.** $\dfrac{u\cos^2v-v^2\sin u\cos u}{\sqrt{u^2\cos^2v+v^2\cos^2u}}$; $\dfrac{v\cos^2u-u^2\sin v\cos v}{\sqrt{u^2\cos^2v+v^2\cos^2u}}$ **59.** $\dfrac{2}{u}$; $\dfrac{4\sinh v\cosh v}{\sinh^2v+\cosh^2v}$ **60.** $\dfrac{y(2x-y)}{1+x^2y^2(x-y)^2}$; $\dfrac{x(x-2y)}{1+x^2y^2(x-y)^2}$ **62.** $\dfrac{xy^3}{(1-x^2y^2)\sqrt{1-x^2y^2}}$; $\dfrac{x^3y}{(1-x^2y^2)\sqrt{1-x^2y^2}}$ $\dfrac{1}{(1-x^2y^2)\sqrt{1-x^2y^2}}$
63. $e^{xyz}(1+3xyz+x^2y^2z^2)$ **64.** 0; $\dfrac{1}{4}$; $-\dfrac{3}{8}$ **65.** $-\dfrac{4}{25}$; $\dfrac{3}{25}$; $\dfrac{4}{25}$ **66.** 0 **69.** $\left(2;-2\dfrac{1}{4}\right)$ **70.** $\left(-\dfrac{1}{3};\dfrac{3}{4}\right)$; $\left(\dfrac{7}{3};-\dfrac{3}{4}\right)$ **71.** $\left(\dfrac{1}{2};\dfrac{1}{2};-\dfrac{1}{2}\right)$
72. $-\dfrac{15}{4}\sqrt{\dfrac{3}{13}}$ **73.** $-\dfrac{30}{13}$ **87.** $2+\sqrt{2}$ **75.** 0,4; **76.**

$\dfrac{3\sqrt{5}}{4}$; **77.** $\dfrac{\sqrt{2}}{2}$ **78.** $(0;-1;0)$ **79.** $\operatorname{div}\overrightarrow{F}=\dfrac{1}{y}+\dfrac{1}{z}+\dfrac{1}{x}$; $\operatorname{rot}\overrightarrow{F}=\left(\dfrac{y}{z^2};\dfrac{z}{x^2};\dfrac{x}{y^2}\right)$ **80.** $\operatorname{div}\overrightarrow{F}=\dfrac{2x}{x^2-y^2}-\dfrac{1}{1+(z-y)^2}+xy$; $\operatorname{rot}\overrightarrow{F}=\left(xz-\dfrac{1}{1+(z-y)^2};-yz;\dfrac{2y}{x^2-y^2}\right)$ **81.** $\operatorname{div}\overrightarrow{F}=-\dfrac{3}{(x+y-z)^2}$; $\operatorname{rot}\overrightarrow{F}=\overrightarrow{\Theta}$ **82.** $\operatorname{div}\overrightarrow{F}=0$; $\operatorname{rot}\overrightarrow{F}=(2x;2x;2y-2z)$ **83.** Local minimum at $\left(-\dfrac{17}{143};\dfrac{7}{143}\right)$ **84.** Local maximum at $(6;4)$; $z_{max}=6912$ **85.** Local minimum at $\left(\dfrac{1}{\sqrt[3]{3}};\dfrac{1}{\sqrt[3]{3}}\right)$; $z_{min}=3\sqrt[3]{3}$ **86.** There is no local extremum at $(-2;0)$; local minimum at $(0;0)$ **87.** There is no local extremum at $(0;0)$, local minimum at $(1;1)$ **88.** $z(1;1)=2$ **89.** $\left(-\dfrac{1}{2}\arctan\dfrac{b}{a};\dfrac{\pi}{4}-\dfrac{1}{2}\arctan\dfrac{b}{a}\right)$ **90.** $\left(\dfrac{1}{3};\dfrac{1}{3};\dfrac{1}{3}\right)$ **91.** $\dfrac{1}{3}$, $\dfrac{1}{3}$ ja $\dfrac{1}{3}$ m **92.** $\dfrac{8}{3}$ **93.** $\ln\dfrac{25}{24}$ **94.** $3\dfrac{3}{4}$ **95.** $\dfrac{19}{12}$. **96.** $\dfrac{6}{35}$ **97.** $\displaystyle\int_{-1}^{1}dx\int_{0}^{1-x^2}f(x;y)dy$ **98.** $\displaystyle\int_{0}^{a}dy\int_{y}^{y+2a}f(x;y)dx$ **99.** $\displaystyle\int_{-1}^{1}dx\int_{x^2}^{\frac{2}{1+x^2}}f(x;y)dy$

100. $\displaystyle\int_{0}^{1}dy\int_{y^2}^{\sqrt[3]{y}}f(x;y)dx$ **101.** $\displaystyle\int_{-1}^{0}dy\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}f(x;y)dx+\int_{0}^{2}dy\int_{y-1}^{1}f(x;y)dx$

102. $\displaystyle\int_{-2}^{2}dx\int_{-\sqrt{x+2}}^{\sqrt{x+2}}f(x;y)dy-\int_{0}^{2}dx\int_{-\sqrt{2x}}^{\sqrt{2x}}f(x;y)dy$

103. $\displaystyle\int_{0}^{1}dy\int_{-\sqrt{y}}^{\sqrt{y}}f(x;y)dx+\int_{1}^{3}dy\int_{-1}^{1}f(x;y)dx-\int_{2}^{3}dy\int_{-\sqrt{y-2}}^{\sqrt{y-2}}f(x;y)dx$ **104.** $\displaystyle\int_{0}^{1}dx\int_{x-1}^{x^2}f(x;y)dy$ **105.** $-10\dfrac{7}{20}$ **106.** $\dfrac{4}{3}a^4$ **107.** $\dfrac{1}{4}$ **108.** e

109. $\displaystyle\int_{0}^{\pi}d\varphi\int_{1}^{2}f(\rho\cos\varphi;\rho\sin\varphi)\rho d\rho$ **110.** $\displaystyle\int_{\frac{\pi}{4}}^{\arctan 2}d\varphi\int_{4\cos\varphi}^{8\cos\varphi}f(\rho\cos\varphi;\rho\sin\varphi)\rho d\rho$

111. $\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} d\varphi \int\limits_{\frac{R}{2\sin\varphi}}^{2R\sin\varphi} f(\rho\cos\varphi;\rho\sin\varphi)\rho d\rho$ **112.** $\frac{\pi}{4}(e^{a^2}-1)$ **113.** π **114.** $\frac{\pi}{4}\left[(1+R^2)\ln(1+R^2)-R^2\right]$ **115.** $\frac{R^3}{3}\left(\pi-\frac{4}{3}\right)$ **116.** $\frac{2\sqrt{2}}{15}a^4$ **117.** $\frac{a^{11}}{110}$ **118.** $\frac{1}{2}\ln 2-\frac{5}{16}$ **119.** $\frac{1}{96}$ **120.** $\int\limits_0^{\frac{\pi}{2}} d\varphi \int\limits_0^2 d\rho \int\limits_0^{\rho^2} f(\rho\cos\varphi;\rho\sin\varphi;z)\rho dz$ **121.** $\frac{4}{3}\pi$ **122.** 8 **123.** $\frac{1}{2}(15-16\ln 2)$ **124.** $a^2(\pi-1)$ **125.** $\frac{16}{3}$ **1126.** 27 **127.** $\frac{3}{35}$ **128.** $\frac{19}{6}\pi$ **129.** $\frac{21\pi}{16}$ **130.** $42\sqrt{2}$ **131.** $\frac{256}{15}a^3$ **132.** $2\pi\sqrt{a^2+b^2}(a^2+\pi b)$ **133.** $\frac{R^4\sqrt{3}}{32}$ **134.** $\ln 2$ **135.** $\frac{\pi}{2}-2$ **136.** $-\frac{a^2+b^2}{2}$ **137.** $\frac{3\pi ab}{16}$ **138.** $\frac{\pi^2}{24}+\frac{1-\sqrt{3}}{2}-\frac{1}{2}\ln 3$ **139.** $3\sqrt{3}$ **140.** 0 **141.** $\iint\limits_D (x^3+y^3)dxdy$ **142.** $\iint\limits_D e^x y dxdy$ **143.** $\frac{2}{3}$ **144.** 4π **145.** 0 **146.** $\frac{25\pi}{2}$ **147.** 4 **148.** 8 **149.** $\sqrt{2}$ **150.** $\frac{\sqrt{21}}{3}$ **151.** $\frac{\pi\sqrt{2}}{2}$ **152.** πR^3 **153.** $\frac{3\pi}{2}$ **154.** $\frac{1}{2}$ **155.** $\frac{2\pi R^7}{105}$ **156.** 0 **157.** $-\frac{1}{8}$ **158.** $\left(\frac{k}{3k-1}\right)^{2k-1}$ **159.** $(-1)^k\frac{k}{6k-5}$ **160.** $S_n=1-\frac{1}{n+1}$; $S=1$ **161.** $S_n=\frac{1}{9}-\frac{1}{9(n+1)}$; $S=\frac{1}{9}$ **162**. $1-\sqrt{2}+\sqrt{n+2}-\sqrt{n+1}$; $1-\sqrt{2}$ **163.** Convergent. **164.** Divergent. **165.** Convergent. **166.** Convergent. **167.** Convergent. **168.** Convergent. **169.** Convergent. **170.** Convergent. **171.** Divergent. **172.** Convergent. **173.** Convergent. **174.** Convergent. **175.** Absolutely onvergent. **176.**Absolutely onvergent. **177.** Conditionally onvergent. **178.** $R=1$; $-1\le x\le 1$ **179.** $R=1$; $-1\le x<1$ **180.** $R=3$; $-1<x<5$ **181.** $-\infty<x<\infty$ **179.** **182.** $\frac{1}{10}-\frac{x}{100}+\frac{x^2}{10^3}-\frac{x^3}{10^4}+\ldots$, converges if $-10<x<10$ **183.** $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots$, converges if $-\infty<x<\infty$ **184.** $1-x^2+x^4-x^6+\ldots$, converges if $-1<x<1$ **185.**

$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$, converges if $-\infty < x < \infty$ **186.** $1 + \frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}$, converges if $-\infty < x < \infty$ **187.** $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$, converges if $-1 \le x \le 1$ **188.** $\frac{4}{\pi}\sum_{k=1}^{\infty} \frac{\sin(2k+1)x}{2k+1}$ **189.** $2\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \sin kx$ **190.** $\frac{\pi^2}{3} + 4\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} \cos kx$ **191.** $\frac{2 \sin a\pi}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^k k}{a^2 - k^2} \sin kx$ **192.** $\sum_{k=1}^{\infty} \frac{\sin kx}{k}$ **193.** $\frac{1}{i\omega}$ **194.** $\frac{2A}{\omega} \sin 2\omega$ **195.** $\frac{2a}{a^2 + \omega^2}$ **196.** $\frac{2ia \sin \frac{\pi\omega}{a}}{\omega^2 - a^2}$