0.1 Töö

On antud juhusliku vektori (X, Y) lähtevalim

x_i	6. 2	4. 4	5. 2	3. 6	3. 9	5. 0,	2. 4	4. 6	4. 0	5. 0
y_i	6. 5	4. 3	5. 4	5. 7	4. 5	7. 1	5. 2	4. 4	4. 5	5. 8

Leidke:
1) lähtevalimi põhjal Teie töö jaoks valim, võttes

$$\begin{aligned} x_{2i+1(Teie)} &= x_{2i+1} + 0.05k,\ x_{2i(Teie)} = x_{2i}, \\ y_{2i+1(Teie)} &= y_{2i+1},\ y_{2i(Teie)} = y_{2i} - 0.01k, \end{aligned}$$

kus $k = m+1-10\cdot[m/10]$ ja m on Teie järjekorranumber harjutustunni nimekirjas ning $[m/10]$ on arvu $m/10$ täisosa;
2) arvkarakteristikute EX, EY, DX, DY, σ_x, σ_y, K$_{x,y}$, $r_{x,y}$ nihketa hinnangud $\overline{x}$, $\overline{y}$, s_x^2, s_y^2, s_x, s_y, K$^*_{x,y}$, $r^*_{x,y}$;
3) regressioonisirge $y = ax + b$ ja regressiooniparabool $y = ax^2 + bx + c$ (vähimruutude mõttes);
4) keskväärtuse EX usaldusvahemik l_β usaldusnivool $\beta = 0.9$;
5) keskväärtuse EX usaldusvahemik usaldusnivool $\beta = 0.9$, kui on teada lisaks, et X allub normaaljaotusele;
6) DX usaldusvahemik usaldusnivool $\beta = 0.95$, kui lisaks on teada, et X allub normaaljaotusele.
Skitseerige:
7) korrelatsiooniväli, st kahe tunnuse X ja Y ühisjaotuse graafiline esitus xy-tasandil ja regressioonisirge ja regressiooniparabooli graafikud.
Kontrollige:
8) hüpoteesi $H_0:\ EX = EY$, võttes $\sigma_x \approx s_x$ ja $\sigma_y \approx s_y$, olulisuse nivool 0.05, kusjuures $H_1:\ EX \neq EY$;
9) hüpoteesi $H_0:$ DX = DY olulisuse nivool 0.1, kusjuures X ja Y on normaaljaotusega ja $H_1:$ D$X \neq$ DY.

Lahendus lähtevalimi korral
2) Leiame

$$\overline{x} \overset{(5.5.5)}{=} \frac{1}{10}\sum_{i=1}^{10} x_i = 4.43,\ \overline{y} \overset{(5.5.5)}{=} \frac{1}{10}\sum_{i=1}^{10} y_i = 5.34,$$

$$\begin{aligned} s_x^2 &\overset{(5.2.15)}{=} \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\overline{x}^2\right) \approx 1.076, \\ s_y^2 &\overset{(5.2.15)}{=} \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - n\overline{y}^2\right) \approx 0.909\,, \end{aligned}$$

$$s_x = \sqrt{s_x^2} \approx \sqrt{1.076} \approx 1.037\,,\ s_y = \sqrt{s_y^2} \approx \sqrt{0.909} \approx 0.953,$$

$$\overline{xy} \overset{(5.5.5)}{=} \frac{1}{10}\sum_{i=1}^{10} x_i y_i = 24.059,$$

$$\mathrm{K}^*_{x,\,y} \overset{(5.2.28)}{=} \frac{n}{n-1}\mathrm{K}^{**}_{x,\,y} \overset{(5.2.29)}{=} \frac{n}{n-1}\left(\overline{xy} - \overline{x}\,\overline{y}\right) \approx 0.448,$$

$$\mathrm{r}^*_{xy} \overset{(5.2.31)}{=} \frac{\mathrm{K}^*_{x,\,y}}{s_x\ s_y} \approx 0.453.$$

3) Regressioonisirge $y = c_1 + c_2 x$ parameetrid c_1 ja c_2 leiame süsteemist (5.5.4), kus

$$\overline{x^2} \overset{(5.5.5)}{=} \frac{1}{10}\sum_{i=1}^{10} x_i^2 \ = 20.593,\ \overline{xy} = 24.059,$$

st süsteemist

$$\begin{cases} c_1 & + & 4.43c_2 & = & 5.34 \\ 4.43c_1 & + & 20.593c_2 & = & 24.059, \end{cases}$$

millest $c_1 \approx 3.497$, $c_2 \approx 0.416$ ja $y = 3.497 + 0.416\,x$. Regressiooniparabooli $y = c_1 + c_2 x + c_3 x^2$ parameetrid c_1, c_2 ja c_3 leiame süsteemist (5.5.6), kus

$$\overline{x^3} \overset{(5.5.7)}{=} \frac{1}{10}\sum_{i=1}^{10} x_i^3 \ = 99.526,\ \overline{x^4} = \frac{1}{10}\sum_{i=1}^{10} x_i^4 = 496.98,$$

$$\overline{x^2 y} \overset{(5.5.8)}{=} \frac{1}{10}\sum_{i=1}^{10} x_i^2 y_i = 113.9,$$

st süsteemist

$$\begin{cases} c_1 & + & 4.43c_2 & + & 20.593c_3 & = & 5.34 \\ 4.43c_1 & + & 20.593c_2 & + & 99.526c_3 & = & 24.059 \\ 20.593c_1 & + & 99.526c_2 & + & 496.98c_3 & = & 113.9, \end{cases}$$

millest $c_1 \approx 8.227$, $c_2 \approx -1.917$, $c_3 \approx 0.272$, $y = 8.227 - 1.917x + 0.272\,x^2$.
4) Leiame

$$\varepsilon_\beta \overset{(5.3.3)}{\approx} \frac{s_x}{\sqrt{n}}\Phi^{-1}\left(\frac{\beta}{2}\right) \overset{\beta=0.9}{\approx} \frac{1.037}{\sqrt{10}}\Phi^{-1}\left(\frac{0.9}{2}\right) \approx \frac{1.037}{\sqrt{10}}1.645 \approx 0.54,$$

$$l_\beta \overset{(5.3.4)}{\approx} \left(\overline{x} - \frac{s}{\sqrt{n}}\Phi^{-1}\left(\frac{\beta}{2}\right),\ \overline{x} + \frac{s}{\sqrt{n}}\Phi^{-1}\left(\frac{\beta}{2}\right)\right) \overset{\beta=0.9}{\approx} (3.89; 4.97),$$

st $P\left(\mathrm{E}X \in (3.89; 4.97)\right) \approx 0.9$.
5) Kuna

$$2\int_0^{t_{0.9;\,9}} f_{T_9}(t)dt = 0.9 \overset{\text{Lisa 4}}{\Rightarrow} t_{0.9;\,9} \approx 1.833,$$

siis

$$\varepsilon_\beta \overset{(5.3.7)}{=} t_\beta s_x/\sqrt{n} \overset{\beta=0.9}{\approx} 1.833 \cdot 1.037/\sqrt{10} \approx 0.60$$

ja

$$l_\beta \overset{(5.3.8)}{=} \left(\overline{x} - t_\beta s/\sqrt{n},\, \overline{x} + t_\beta s/\sqrt{n}\right) \overset{\beta=0.9}{\approx} (3.83; 5.03)\,.$$

st

$$P\left(3.83 < \mathrm{E}X < 5.03\right) \overset{(5.3.9)}{\approx} 0.9.$$

6) Kuna

$$P\left(Y_9 < v_1\right) = \frac{1-0.95}{2} = 0.025 \overset{\text{Lisa 3}}{\Rightarrow} v_1 \approx 2.70\,,$$

$$D_2 \overset{(5.3.13)}{=} (n-1)\, s_x^2/v_1 \approx 9 \cdot 1.076/2.70 \approx 3.59,$$

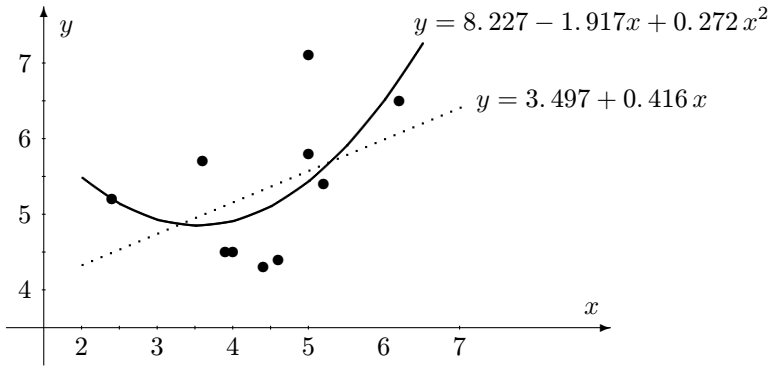
$$P\left(Y_9 < v_2\right) = \frac{1+\beta}{2} \overset{\text{Lisa 3}}{\Rightarrow} v_2 \approx 19.02,$$

$$D_1 \overset{(5.3.13)}{=} (n-1)\, s^2/v_2 \approx 9 \cdot 1.076/19.02 \approx 0.51,$$

siis

$$P(0.51 < \mathrm{D}X < 3.59) \approx 0.95.$$

7)



8) Leiame

$$\theta^* \overset{(5.4.8)}{=} \frac{\overline{x} - \overline{y}}{\sqrt{\sigma_X^2/n + \sigma_Y^2/m}} \approx \frac{\overline{x} - \overline{y}}{\sqrt{s_X^2/n + s_Y^2/m}} \approx$$

$$\approx \frac{4.43 - 5.34}{\sqrt{1.076/10 + 0.909/10}} \approx -2.04,$$

$$\Phi(\theta_{kr}) \overset{(5.4.8)}{=} 0.5 - \alpha/2 = 0.5 - 0.1/2 = 0.45 \overset{\text{Lisa 2}}{\Rightarrow} \theta_{kr} \approx 1.645.$$

Et $|\theta^*| > \theta_{kr}$, siis lükkame hüpoteesi H_0 tagasi.

9) Kuna

$$\begin{aligned} z^* \overset{(5.4.14)}{=} s_x^2/s_y^2 &\approx 1.076/0.909 \approx 1.184, \\ F_{Z_{\hat{n}_1,\,\hat{n}_2}}(z_{krv}) &= \alpha/2 = 0.05 \overset{(5.4.17)}{\Rightarrow} z_{krv} \approx 0.315, \\ F_{Z_{\hat{n}_1,\,\hat{n}_2}}(z_{krp}) &= 1 - \alpha/2 = 0.95 \overset{(5.4.18)}{\Rightarrow} z_{krp} \approx 3.179, \end{aligned}$$

siis $z_{krv} \le z^* \le z_{krp}$, st loeme hüpoteesi H_0 katsetulemustega kooskõlas olevaks.